Tutorial 3: Texture Mapping

This tutorial concerns texture mapping onto polygons. We will consider the case of a terminal in which individual pixels can be set to any intensity in the range $[0, I_{max}]$.

Suppose that the following function is used to transform texture coordinates (α, β) into an intensity value:

$$I = \frac{(\alpha + \beta)}{2} I_{max}$$

I is therefore the intensity given to a pixel which corresponds to position (α, β) in texture space.

A quadrilateral is projected onto the screen. The screen coordinates and texture coordinates of its vertices are as follows:

Vertex	Screen coordinates (x, y)	Texture coordinates (α, β)
$\mathbf{P}_1$	(5,5)	(0, 0)
$\mathbf{P}_2$	(15,30)	(0, 1)
$\mathbf{P}_3$	(50,50)	(1, 1)
$\mathbf{P}_4$	(40,10)	(1, 0)
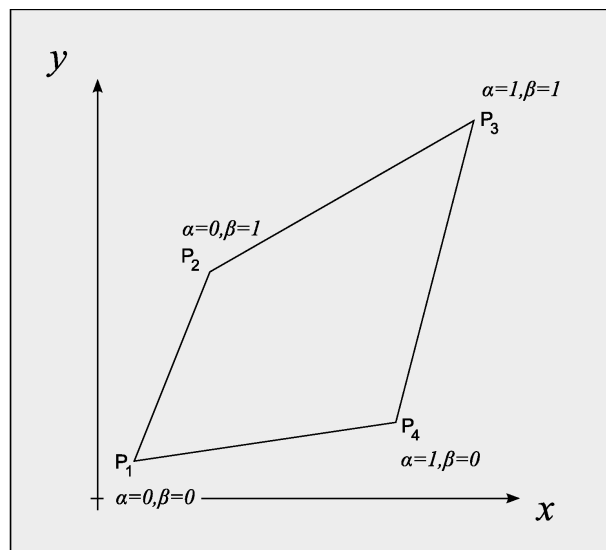
1. Let $\mathbf{P}_t$ be the point with screen coordinates $(30, 30)$. Calculate the texture coordinates (α, β) corresponding to $\mathbf{P}_t$ and find the intensity to be applied to $\mathbf{P}_t$.

N.B. Use bi-linear interpolation to find the texture coordinates, i.e. solve for (α, β) the equation:

$$\mathbf{p} = \alpha\beta(\mathbf{c} - \mathbf{b}) + \alpha\,\mathbf{a} + \beta\,\mathbf{b}$$

Where

$$\mathbf{p} = \mathbf{P}_t - \mathbf{P}_1 \qquad \mathbf{a} = \mathbf{P}_4 - \mathbf{P}_1 \qquad \mathbf{b} = \mathbf{P}_2 - \mathbf{P}_1 \qquad \mathbf{c} = \mathbf{P}_3 - \mathbf{P}_4$$

Computing texture coordinates for every pixel by using bi-linear interpolation is very time consuming, since the solution of a quadratic is required. By doing the calculations differentially, much computation time can be saved.

For example, consider the line from $\mathbf{P}_1$ to $\mathbf{P}_2$: At $\mathbf{P}_1$ $\beta = 0$ and at $\mathbf{P}_2$ $\beta = 1$ and there are 26 pixels on the line (the biggest change in the screen coordinates is along the *y*-component, from 5 to 30, which gives 26 pixels including the end-points).

Moving along the line from $\mathbf{P}_1$ to $\mathbf{P}_2$, the differential change in β from one pixel to the next is $\frac{1}{25}$ because 26 pixels (including end-points) have 25 spaces between them.

Thus, we can estimate the values of β at each pixel along the line simply by adding the differential as we move from one pixel to its neighbour.

2. What are the differentials in α and β for the four lines bounding the quadrilateral?

3. Now consider the horizontal line through the point $(30, 30)$. Using the differential method above, find the values of α and β at the points where it intersects the sides of the quadrilateral, and hence find the differentials in α and β along the line.

4. Use these differential values to compute α and β at the point $(30, 30)$. Find also the corresponding intensity to be given to the pixel. Compare your results with the values obtained in part 1.

5. Can you suggest why the methods in parts 1 and 3 do not give exactly the same result?

Notice that with the differential method, calculating α and β for most of the pixels in the quadrilateral requires only two additions, and hence is much faster.