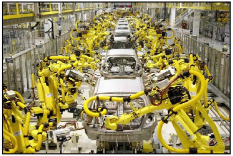
Distributed control and game design

From strategic agents to programmable machines

Dario Paccagnan

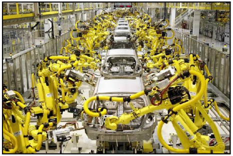
PhD Defense

Coordination of multiagent systems



Coordination of multiagent systems

Competitive ←——— - - - - ———→ Cooperative



PhD research overview

Aggregative games

- Large population, algorithms
  [TAC18a]
- Equilibrium efficiency
  [L-CSS18], [CDC18]
- Algorithms and applications
  [CDC16], [ECC16], [CPS18]
- Traffic and Inertial equilibria
  [IFAC17], [CDC17]

Combinatorial allocation

- Optimal utility design
  [Submitted, J18a]
  [Submitted, J18b]
- Role of information
  [TAC18b]
  [Allerton17]
- Worst vs best perf. tradeoff
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Others [CDC15], [PLANS14]

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Aggregative games

- Introduction
- Convergence between Nash and Wardrop
- Efficiency of equilibria

Aggregative games

players: $i \in \{1, \ldots, M\}$

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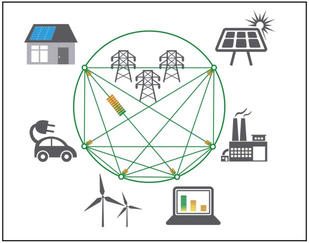

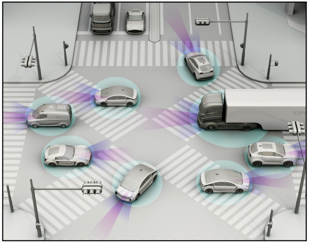

Two equilibrium notions

Two equilibrium notions

$\hat{x}$ Nash equilibrium

$$J^i(\hat{x}^i, \sigma(\hat{x})) \leq J^i(x^i, \sigma(x^i, \hat{x}^{-i}))$$

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What is the relation between $\hat{x}$ and $\bar{x}$?

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Wardrop operator

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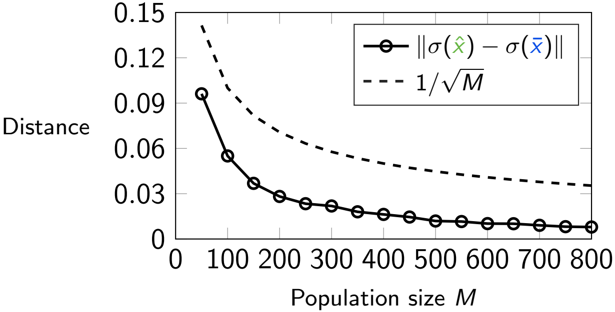

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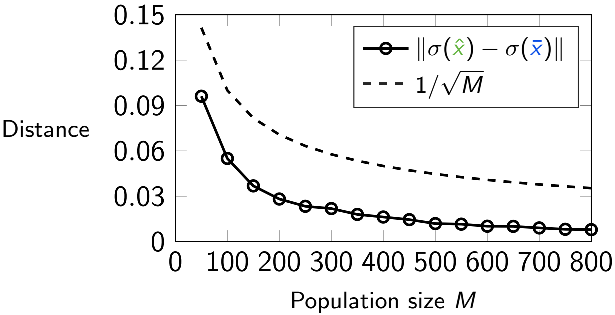

Consequences:

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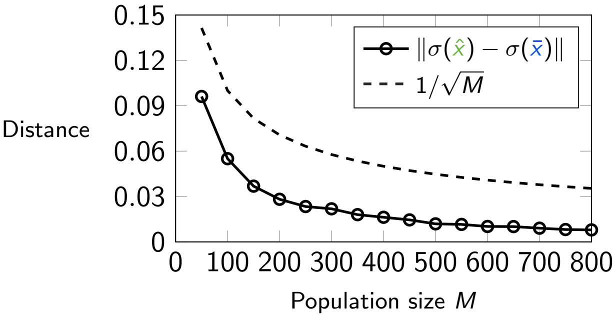

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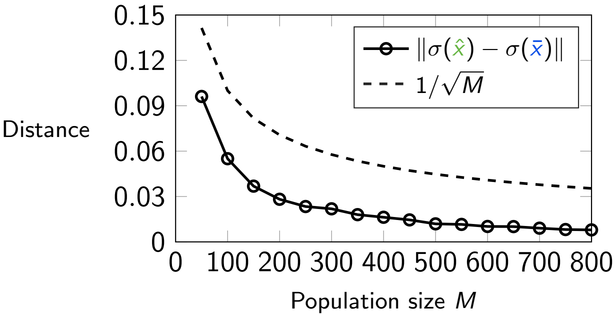

Consequences:
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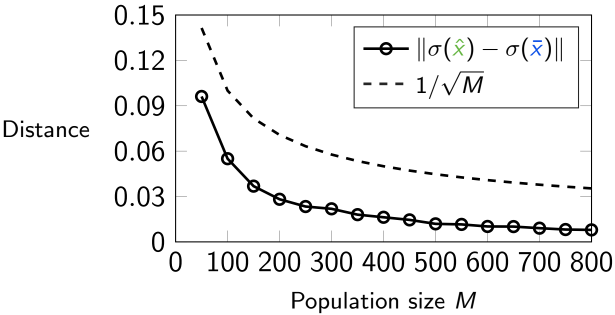

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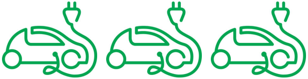
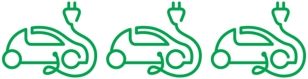


- A fleet of EVs to recharge

Equilibrium efficiency: electric vehicle charging



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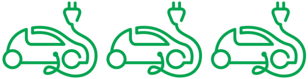

- A fleet of EVs to recharge   players: $i \in \{1, \ldots, M\}$
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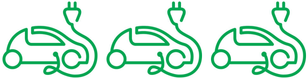

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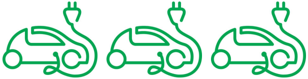

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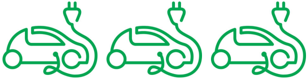

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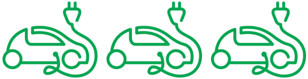

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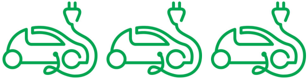
System level objective

- Minimize congestion

Equilibrium efficiency: electric vehicle charging



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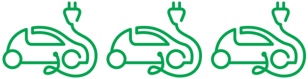

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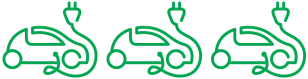
How much does selfish behaviour degrade the performance?

Equilibrium efficiency: electric vehicle charging



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How much does selfish behaviour degrade the performance?

$$\mathrm{PoA} = \frac{\max_{x \in \mathrm{NE(G)}} J_s(x)}{J_s(x_{\mathrm{opt}})} \geq 1$$

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Theorem (Equilibrium efficiency) [L-CSS18]

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Assume sufficient regularity
Assume $p(z+d) = [g(z_1+d_1); \dots; g(z_n+d_n)], \qquad g : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

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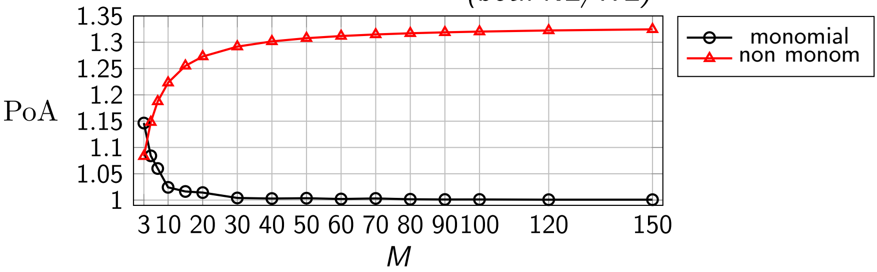

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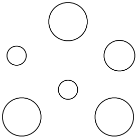
Combinatorial allocation

- Introduction
- GMMC problems are intractable
- Utility design approach and performance guarantees

Combinatorial resource allocation

- a set of resources



Combinatorial resource allocation

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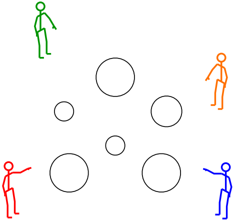
- a set of agents



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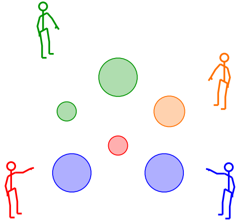

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Goal: assign resources to agents to maximize a given welfare function

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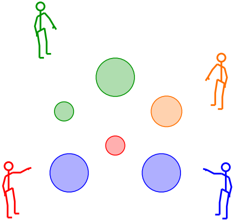

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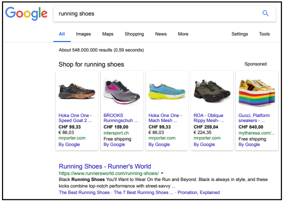

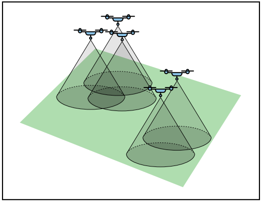

Generalized Multiagent Maximum Coverage (GMMC)

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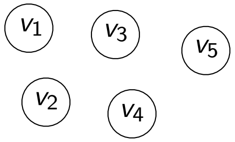

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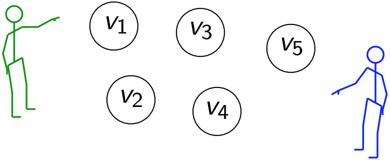

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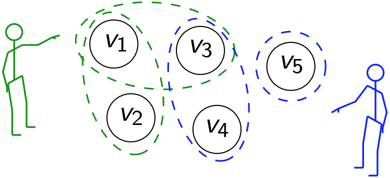

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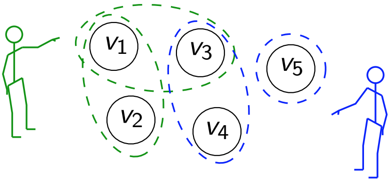

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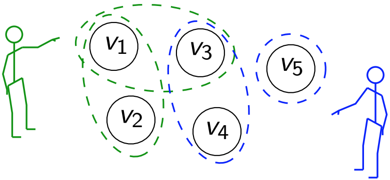

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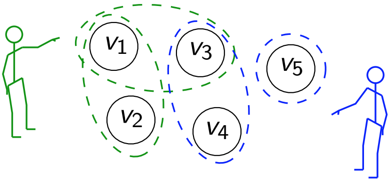

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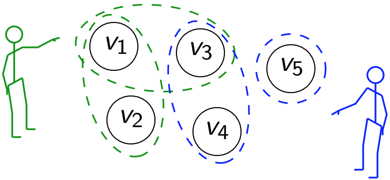

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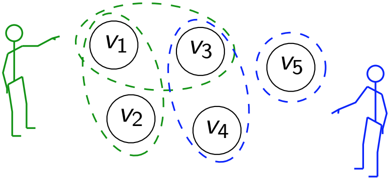

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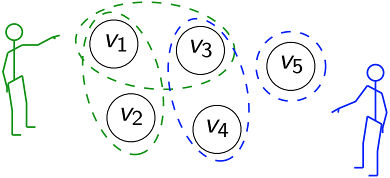

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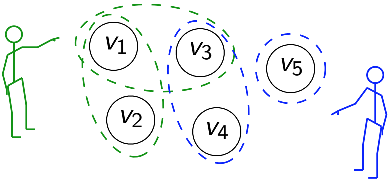

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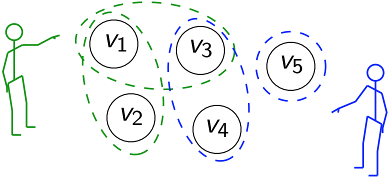

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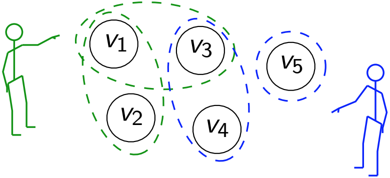

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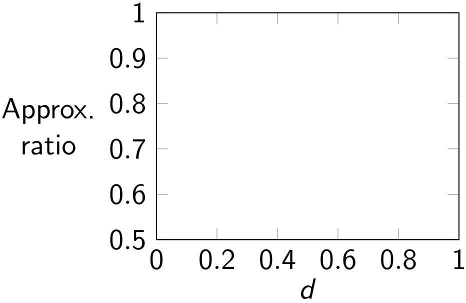

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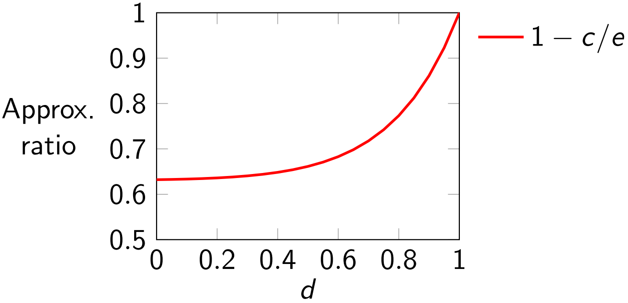

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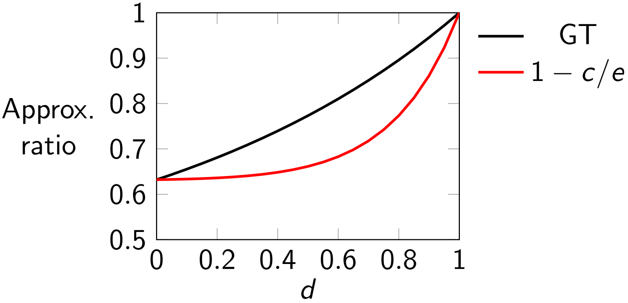

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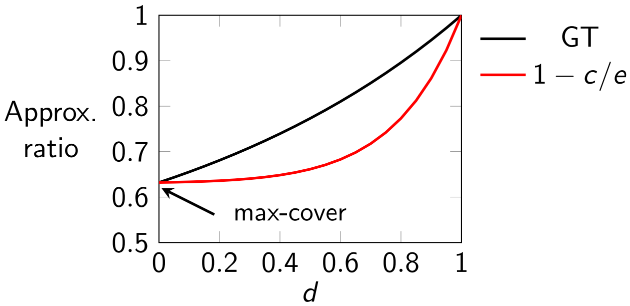

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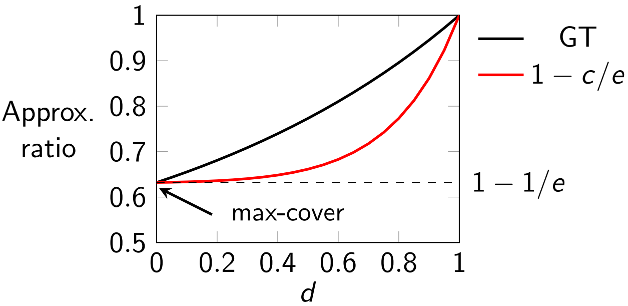

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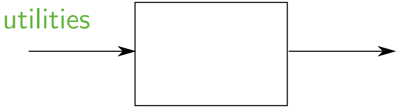

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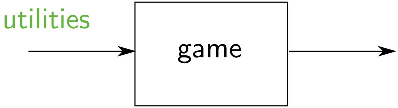

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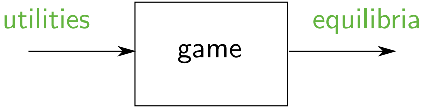

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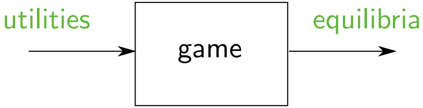

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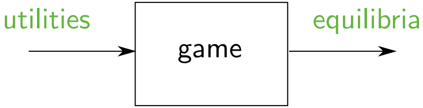

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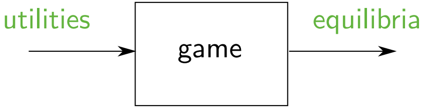

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$u_i(a_i, a_{-i})$

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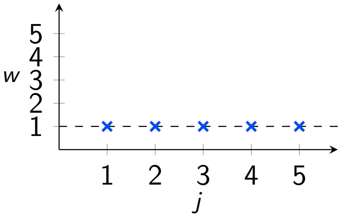

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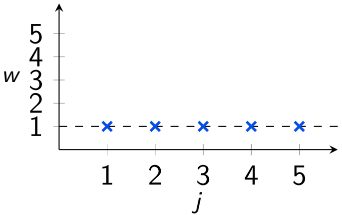

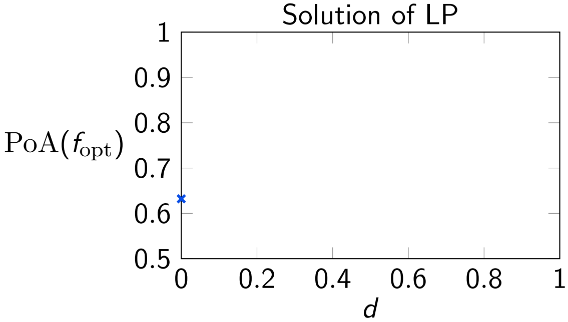

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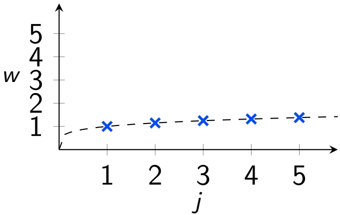
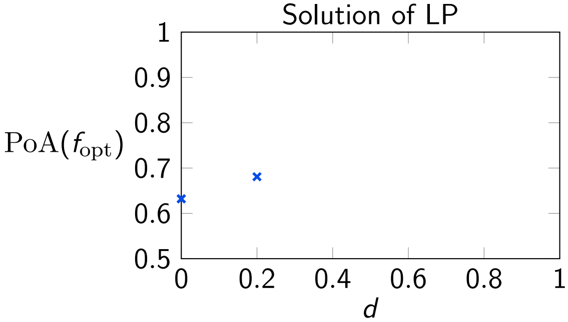
- \# agents ≤ 40
- $w(j) = j^d$, $d = 0.2$

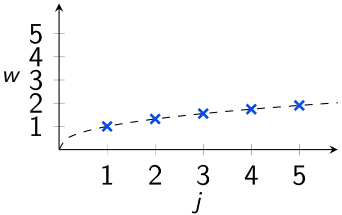


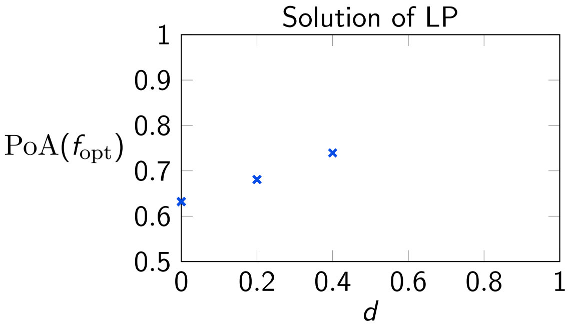


Back to the main result

Example:

- \# agents ≤ 40
- $w(j) = j^d$, $d = 0.4$

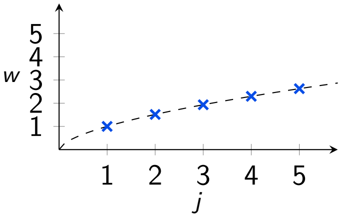


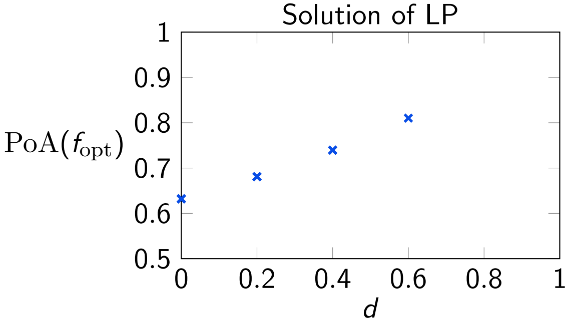


Back to the main result

Example:

- \# agents ≤ 40
- $w(j) = j^d$, $d = 0.6$

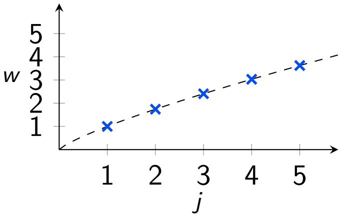


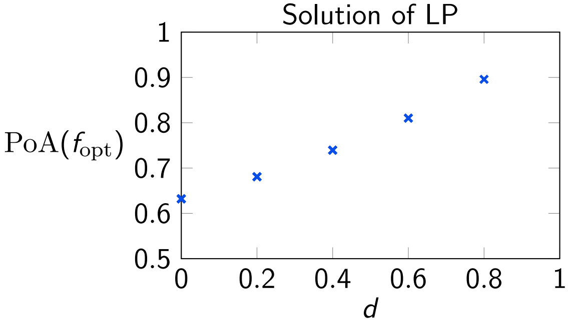


Back to the main result

Example:

- $\#$ agents ≤ 40
- $w(j) = j^d$, $d = 0.8$

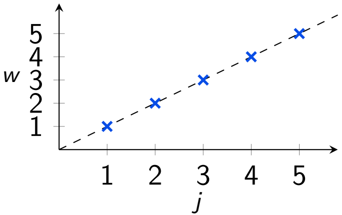


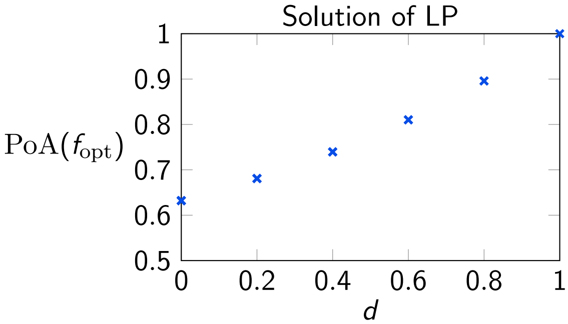


Back to the main result

Example:

- \# agents ≤ 40
- $w(j) = j^d$, $d = 1$

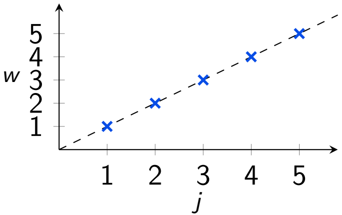


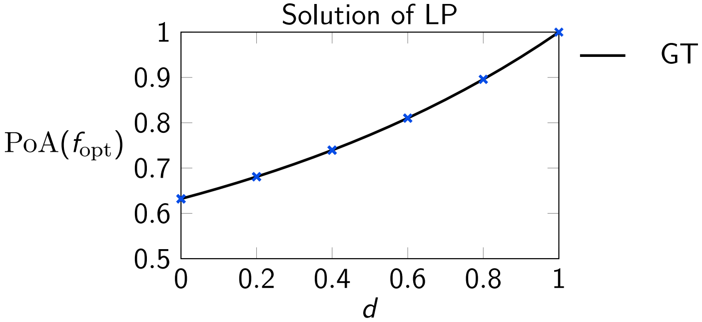


Back to the main result

Example:

- $\#$ agents ≤ 40
- $w(j) = j^d$, $d = 1$

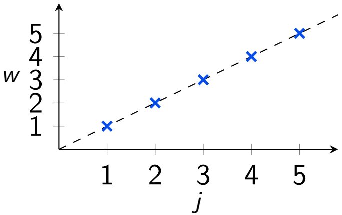


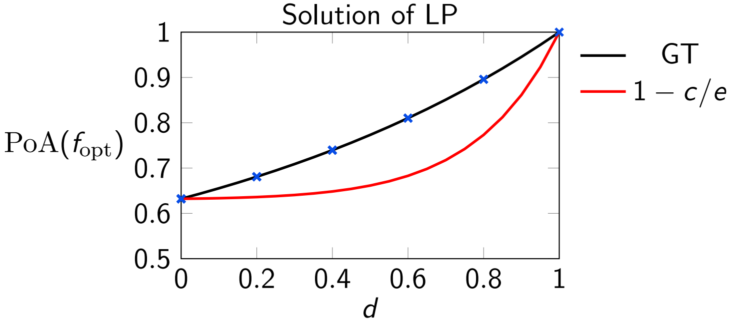


Back to the main result

Example:

- $\#$ agents ≤ 40
- $w(j) = j^d$, $d = 1$





Conclusions and Outlook

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Aggregative games

- Convergence between Nash and Wardrop
- Equilibrium efficiency

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- GMMC problems and utility design approach
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- Submodular maximization

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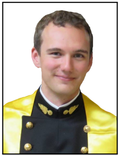
G. Burger


B. Ogunsola

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