QUERY-BASED ENTAILMENT AND INSEPARABILITY FOR $\mathcal{ALC}$ ONTOLOGIES

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Are two ontologies (knowledge bases or TBoxes) **distinguishable** by means of **conjunctive queries** (CQs) or **unions of CQs** (UCQs)?

Modularisation

$\mathcal{O}' \subseteq \mathcal{O}$ is a **Θ-module** of $\mathcal{O}$ if $\mathcal{O}' \equiv_{\Theta}^{\text{(U)CQ}} \mathcal{O}$

Versioning

$\mathcal{O}_2$ is a **Θ-version** of $\mathcal{O}_1$ if $\mathcal{O}_2 \equiv_{\Theta}^{\text{(U)CQ}} \mathcal{O}_1$

$\mathcal{K}_1 = (\{A \sqsubseteq B \sqcup C\}, \{A(a)\})$
$\mathcal{K}_2 = (\emptyset, \{A(a)\})$

$\mathcal{K}_1 \not\equiv_{\{A,B,C\}}^{\text{UCQ}} \mathcal{K}_2$
$\mathcal{K}_1 \equiv_{\{A,B,C\}}^{\text{CQ}} \mathcal{K}_2$
$\mathcal{K}_1 \equiv_{\{A\}}^{\text{UCQ}} \mathcal{K}_2$

Query

Given a signature Σ and $\mathcal{Q} \in \{\text{CQ}, \text{UCQ}\}$, KBs $\mathcal{K}_1 = (\mathcal{T}_1, \mathcal{A}_1)$ and $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ are **Σ-$\mathcal{Q}$-inseparable**, $\mathcal{K}_1 \equiv_{\Sigma}^{\mathcal{Q}} \mathcal{K}_2$, if

$$\mathcal{K}_2 \models \boldsymbol{q}(\boldsymbol{a}) \iff \mathcal{K}_1 \models \boldsymbol{q}(\boldsymbol{a})$$

for all Σ-queries $\boldsymbol{q} \in \mathcal{Q}$ and all individuals $\boldsymbol{a}$ in $\mathcal{K}_1, \mathcal{K}_2$

Inseparability

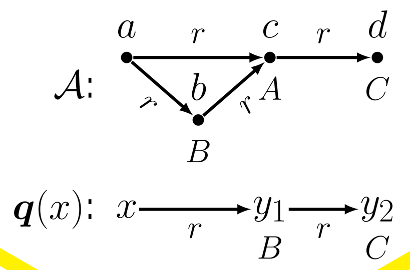
Given signatures Σ_1 and Σ_2, TBoxes $\mathcal{T}_1$ and $\mathcal{T}_2$ are **(Σ_1,Σ_2)-$\mathcal{Q}$-inseparable**, $\mathcal{T}_1 \equiv_{(\Sigma_1,\Sigma_2)}^{\mathcal{Q}} \mathcal{T}_2$, if

$$(\mathcal{T}_1, \mathcal{A}) \equiv_{\Sigma_2}^{\mathcal{Q}} (\mathcal{T}_2, \mathcal{A})$$

for all Σ_1-ABoxes $\mathcal{A}$.

$\mathcal{T}_1 = \{A \sqsubseteq B \sqcup C\}, \mathcal{T}_2 = \emptyset$
$\Sigma = \{R, A, B, C\}$

$\mathcal{T}_1 \not\equiv_{(\Sigma,\Sigma)}^{\text{CQ}} \mathcal{T}_2$, for:



Knowledge Exchange

$\mathcal{K}_2$ in Σ_2 is a **universal (U)CQ-solution** for $\mathcal{K}_1$ in Σ_1 under a mapping $\mathcal{T}_{12}$ from Σ_1 to Σ_2 if $\mathcal{K}_2 \equiv_{\Sigma_2}^{\text{(U)CQ}} \mathcal{K}_1 \cup \mathcal{T}_{12}$

ontology $\mathcal{O}_1$

ontology $\mathcal{O}_2$

signature $\Sigma = \{\text{spots, tail, whiskers}\}$

Forgetting

$\mathcal{O}'$ is the result of **forgetting** Σ in $\mathcal{O}$ if $\text{sig}(\mathcal{O}') \subseteq \text{sig}(\mathcal{O}) \setminus \Sigma$ and $\mathcal{O}' \equiv_{\text{sig}(\mathcal{O})\setminus\Sigma}^{\text{(U)CQ}} \mathcal{O}$

Complexity of inseparability for Horn DLs and (U)CQs

$\mathcal{EL}$: PTime
DL-Lite$_{core}^{\mathcal{H}}$: ExpTime
Horn-$\mathcal{ALC}$: ExpTime
Horn-$\mathcal{ALCHI}$: 2ExpTime

Criteria for KBs

$\mathcal{K}_1$ Σ-UCQ entails $\mathcal{K}_2$ iff

$$\forall \mathcal{I}_1 \models \mathcal{K}_1 \;\; \exists \mathcal{I}_2 \models \mathcal{K}_2 \quad \mathcal{I}_2 \xrightarrow[\text{hom}]{\text{fin}}_{\Sigma} \mathcal{I}_1$$

$\mathcal{K}_1$ Σ-CQ entails $\mathcal{K}_2$ iff

$$\prod_{\mathcal{I}\models\mathcal{K}_2} \mathcal{I} \xrightarrow[\text{hom}]{\text{fin}}_{\Sigma} \prod_{\mathcal{I}\models\mathcal{K}_1} \mathcal{I}$$

For Horn-TBoxes

If there exist Σ_1-ABox $\mathcal{A}$ and Σ_2-CQ $\boldsymbol{q}$ with $(\mathcal{T}_2, \mathcal{A}) \models \boldsymbol{q}$ and $(\mathcal{T}_1, \mathcal{A}) \not\models \boldsymbol{q}$ then there exist *tree-shaped* Σ_1-ABox $\mathcal{A}'$ and *tree-shaped* Σ_2-CQ $\boldsymbol{q}'$ with $(\mathcal{T}_2, \mathcal{A}') \models \boldsymbol{q}'$ and $(\mathcal{T}_1, \mathcal{A}') \not\models \boldsymbol{q}'$

Complexity of inseparability for OWL 2 profiles and (U)CQs

$\mathcal{EL}$: ExpTime
DL-Lite$_{core}^{\mathcal{H}}$: ExpTime

$\mathcal{ALC}$ and (rooted) UCQs

2ExpTime

using two-way alternating automata

$\mathcal{ALC}$ and (rooted) CQs

undecidable

***Horn*-$\mathcal{ALC}$ and (U)CQs**

2ExpTime

ExpTime for rooted queries

using tree automata

unibz

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