Description Logic Knowledge Base Exchange

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Bolzano

Outline

Outline

Knowledge Base Exchange: a Simple Scenario

Category

- ▷ *Item*
  - ▷ *FootWear*
    - ⋆*OpenShoes*
      - ⋆*Strappy*
      - ⋆*Plateau*
      - ⋆*HighHeel*
    - . . .
  - ▷ *Apparel*
  - . . .

Size Choose size▽ **Color** Choose color▽ **Brand** Choose brand▽

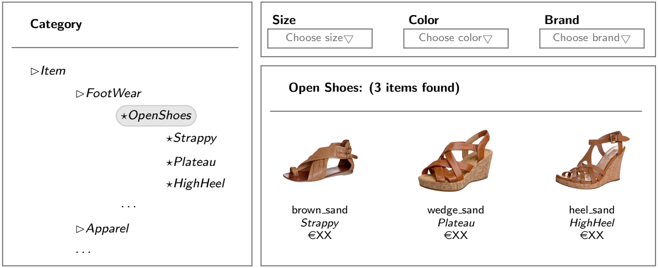
Open Shoes: (3 items found)

brown_sand
Strappy
€XX

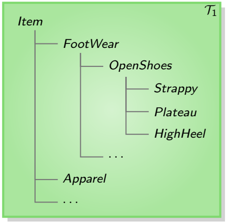
wedge_sand
Plateau
€XX

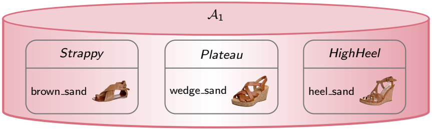
heel_sand
HighHeel
€XX

Knowledge Base Exchange: a Simple Scenario



Viewed as a knowledge base:





Knowledge Base Exchange: a Simple Scenario

The website after **restructuring**:

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Size	**Color**	**Brand**
Choose size▽	Choose color▽	Choose brand▽

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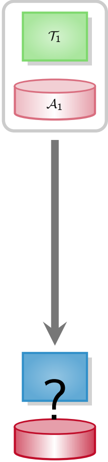
brown_sand	wedge_sand	heel_sand
Strappy	*Plateau*	*HighHeel*
€XX	€XX	€XX



The website after **restructuring**:

Category

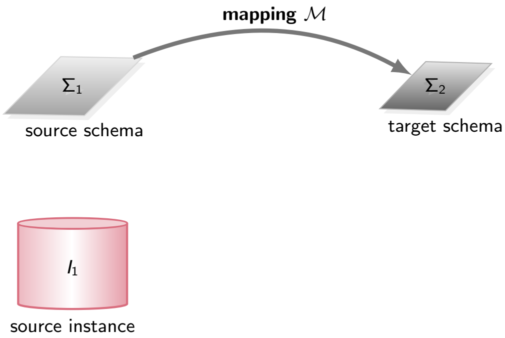
▷ *Product*
- ▷ *Shoes*
  - ⋆*Sandals*
    - ⋆*Classic*
    - ⋆*Platform*
    - ⋆*Heeled*
  - ...
- ▷ *Clothing*
- ...

Size	**Color**	**Brand**
Choose size▽	Choose color▽	Choose brand▽

Sandals: (3 products found)

brown_sand	wedge_sand	heel_sand
Classic	*Platform*	*Heeled*
€XX	€XX	€XX

Data Exchange [Fagin et al., 2003]



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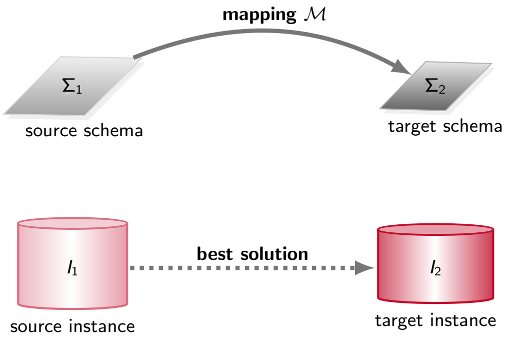

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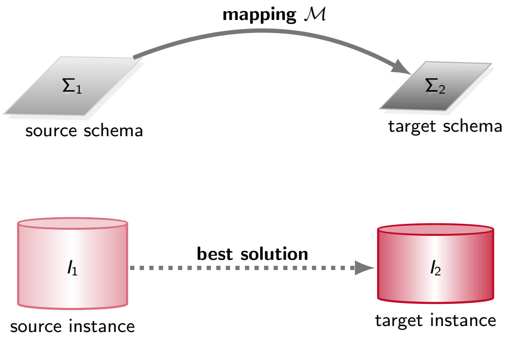

Mapping $\mathcal{M}$ is a set of inclusions of conjunctive queries (CQs)

$$\forall x, y \ \big(q_1(x, y) \rightarrow \exists z \ q_2(x, z)\big).$$

I_1 is a complete database instance.
I_2 is an incomplete database instance.

Data Exchange Example

$$\mathcal{M}: \quad \forall a, t.(\quad AuthorOf(a, t) \quad \rightarrow \quad \exists g.BookInfo(t, a, g) \quad)$$

I_1 :

AuthorOf	
nabokov	lolita
tolkien	lotr

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I_1 :

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nabokov	lolita
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I_2 :

BookInfo		
lolita	nabokov	tragicomedy
lotr	tolkien	fantasy

I_2 is a solution for I_1 under $\mathcal{M}$.

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I_2' :

BookInfo		
lolita	nabokov	null_1
lotr	tolkien	null_2

I_2 is a solution for I_1 under $\mathcal{M}$.
I_2' is a universal solution for I_1 under $\mathcal{M}$.

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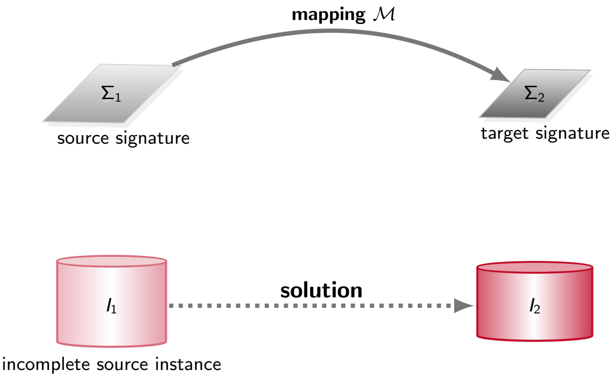
$\Rightarrow$ there is a homomorphism from I_2' to I_2.

Incomplete Data and Knowledge Exchange

A framework for Data Exchange with incomplete data was proposed by Arenas et al. [2011].



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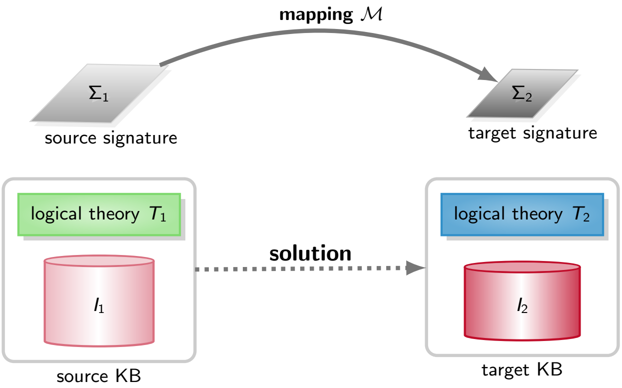

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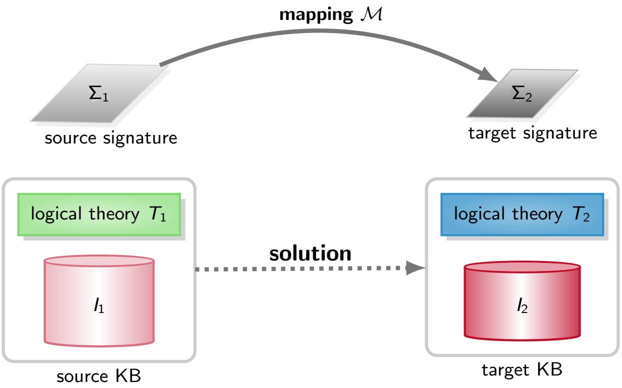

We specialize this framework to Description Logics, and in particular to *DL-Lite*$_{\mathcal{R}}$.

Description Logic $DL\text{-}Lite_{\mathcal{R}}$

Description Logics (DLs) are decidable fragments of First-Order Logic, used as Knowledge Representation formalisms.

DLs talk about the domain of interest by means of

- **concepts** (unary predicates): Author, Book, A, B
- **roles** (binary predicates): AuthorOf, WrittenBy, P, R

$DL\text{-}Lite_{\mathcal{R}}$ is a light-weight DL that asserts

- **concept inclusions** and **disjointness** of *atomic concepts* A, *the domain* $\exists P$ and *the range* $\exists P^-$ of atomic roles P,
  $Book \sqsubseteq \exists WrittenBy$,
- **role inclusions** and **disjointness** of *atomic roles* P and their *inverses* P^-, $AuthorOf \sqsubseteq WrittenBy^-$,
- **ground facts** Author(nabokov), AuthorOf(nabokov,lolita),
  $A(a)$, $P(a, b)$.

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- **ground facts** Author(nabokov), AuthorOf(nabokov,lolita), $A(a)$, $P(a, b)$.

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Satisfiability check over a $DL\text{-}Lite_{\mathcal{R}}$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ can be done in polynomial time (in fact, in NLogSpace).

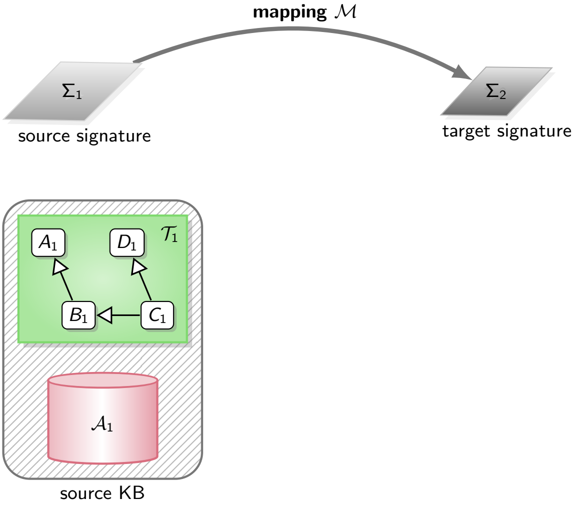
Outline

In this thesis, we

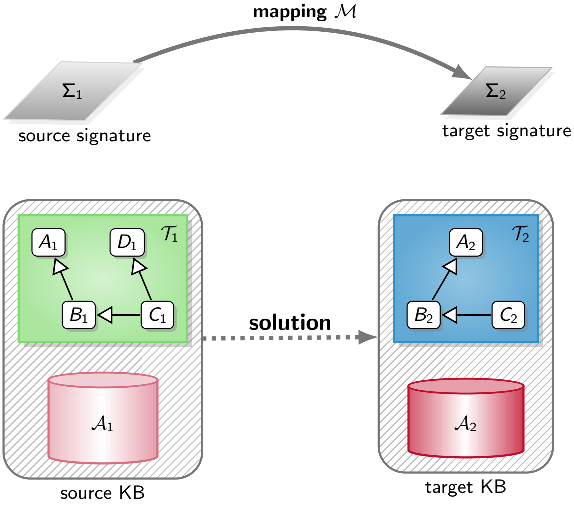
1. Propose a general framework for exchanging Description Logic Knowledge Bases.
2. Define and analyse relevant reasoning problems in this setting.
3. Develop reasoning techniques and characterize the computational complexity of the problems.

Outline

1. Introduction
2. Summary of Work
3. Results
4. Technical Development
   - Universal Solutions
   - Universal UCQ-solutions
   - UCQ-representations


mapping $\mathcal{M}$
Σ_1
source signature
Σ_2
target signature
$\mathcal{T}_1$
A_1
D_1
B_1
C_1
$\mathcal{A}_1$
source KB

1. Knowledge Base Exchange Framework



2. Reasoning Problems

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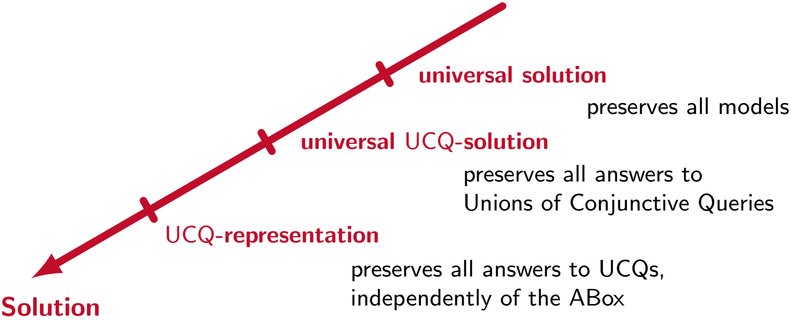
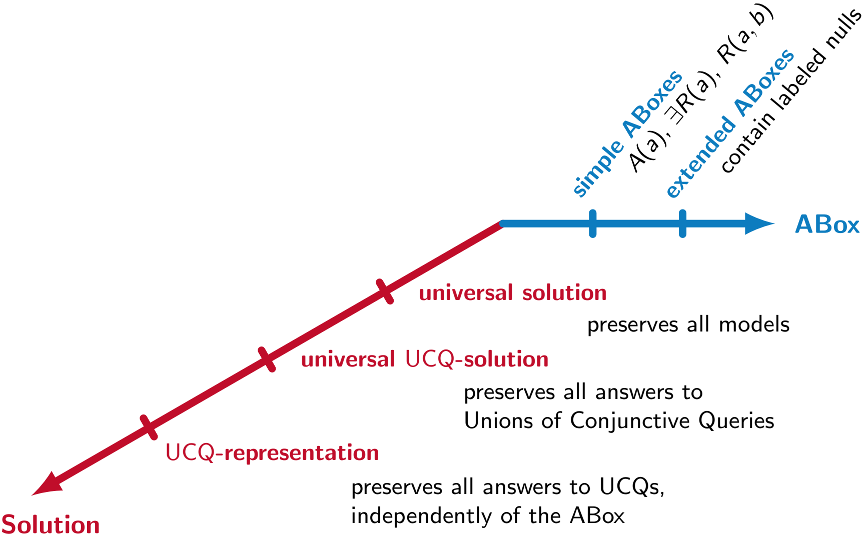
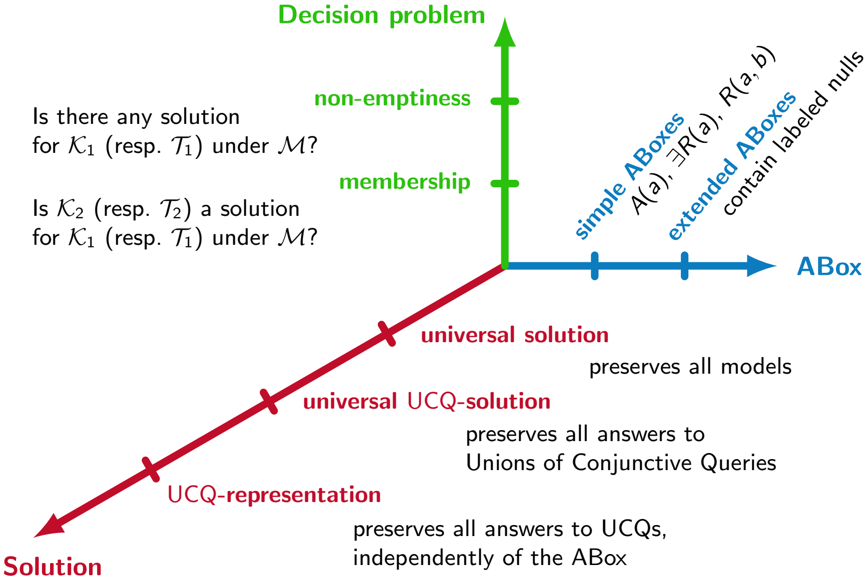



simple ABoxes
$A(a)$, $\exists R(a)$, $R(a, b)$
extended ABoxes
contain labeled nulls
ABox
universal solution
preserves all models
universal UCQ-solution
preserves all answers to
Unions of Conjunctive Queries
UCQ-representation
preserves all answers to UCQs,
independently of the ABox
Solution


Decision problem
non-emptiness
membership
Is there any solution
for $\mathcal{K}_1$ (resp. $\mathcal{T}_1$) under $\mathcal{M}$?
Is $\mathcal{K}_2$ (resp. $\mathcal{T}_2$) a solution
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3. Results

Universal solutions	simple ABoxes	extended ABoxes
Membership	PTIME-complete	NP-complete
Non-emptiness	PTIME-complete	PSPACE-hard, in EXPTIME

Universal UCQ-**solutions**	simple ABoxes	extended ABoxes
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UCQ-**representations**	Complexity
Membership	NLOGSPACE-complete
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❶ games 

circuit value problem

ad-hoc 

3-colorability

❷ automata 

validity of QBF

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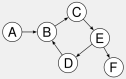
❹ ❺ graph-theoretic



reachability in directed graphs

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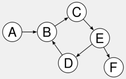
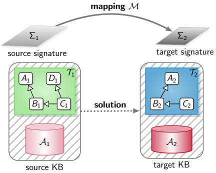
❹ ❺ graph-theoretic 

reachability in directed graphs

Outline

The Essence of Knowledge Base Exchange

A **mapping** is a triple $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$,
where $\mathcal{T}_{12}$ is a set of *DL-Lite*$_\mathcal{R}$ inclusions from Σ_1 to Σ_2



$\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ is a *DL-Lite*$_\mathcal{R}$ knowledge base over Σ_1 (**source** KB)
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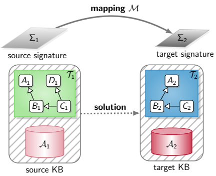

For a KB $\mathcal{K}$, we denote by $\mathcal{U}_{\mathcal{K}}$ the canonical model of $\mathcal{K}$ (chase in databases).

- $\langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff*
  $\mathcal{T}_2 = \emptyset$ and $\mathcal{U}_{\mathcal{A}_2}$ is Σ_2-homomorphically equivalent to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$.

- $\langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal UCQ-solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff
  $\mathcal{U}_{\langle \mathcal{T}_2, \mathcal{A}_2 \rangle}$ is finitely Σ_2-homomorphically equivalent to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$.

- $\mathcal{T}_2$ is a UCQ-representation for $\mathcal{T}_1$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff**
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$\star$ plus one more condition with little technical impact

$\star\star$ plus one more condition for checking consistency

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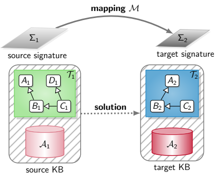

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We present our 5 techniques that check the existence of the homomorphisms.

$\star$ plus one more condition with little technical impact

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Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, where

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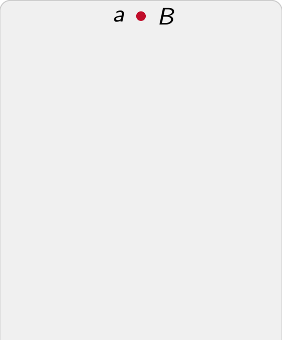
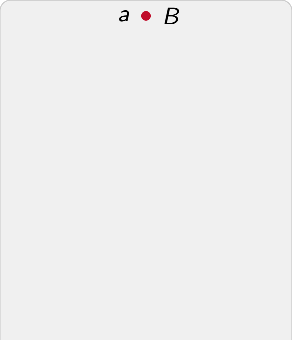


The canonical model $\mathcal{U}_{\mathcal{K}}$



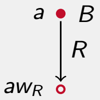
The generating model $\mathcal{G}_{\mathcal{K}}$

We call w_R and w_S *witnesses* of $\mathcal{K}$, denoted $\mathsf{Wit}(\mathcal{K})$.
Moreover, we write, e.g., $a \leadsto_{\mathcal{K}} w_R$, or $w_R \leadsto_{\mathcal{K}} w_S$.

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, where

$\mathcal{A} = \{B(a)\}$

$\mathcal{T} = \{B \sqsubseteq \exists R$



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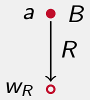

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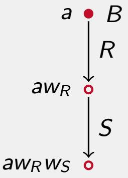
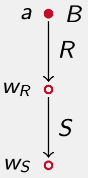
$$\mathcal{T} = \{B \sqsubseteq \exists R \quad \exists R^- \sqsubseteq \exists S$$



The canonical model $\mathcal{U}_\mathcal{K}$



The generating model $\mathcal{G}_\mathcal{K}$

We call w_R and w_S *witnesses* of $\mathcal{K}$, denoted $\mathrm{Wit}(\mathcal{K})$.
Moreover, we write, e.g., $a \rightsquigarrow_\mathcal{K} w_R$, or $w_R \rightsquigarrow_\mathcal{K} w_S$.

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, where

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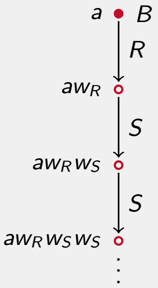

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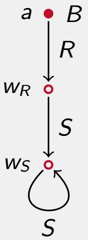

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The Canonical and Generating Models

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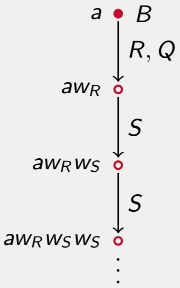
$$R \sqsubseteq Q\}$$



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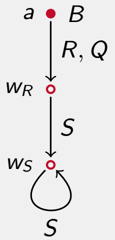

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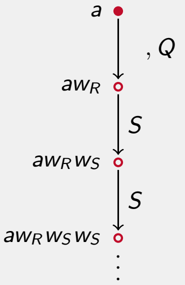
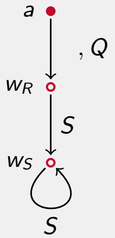
$$\Sigma = \{Q, S\}$$



The canonical model $\mathcal{U}_{\mathcal{K}}{}^{\Sigma}$



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Outline

Membership for Simple Universal Solutions is in PTIME

$\mathcal{A}_2$ is a universal solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff* there exist

- a Σ_2-homomorphism from $\mathcal{U}_{\mathcal{A}_2}$ to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$,
- a Σ_2-homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$.

$\star$ plus one more condition of no technical interest

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- a Σ_2-homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$. via Reachability Games on graphs

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For a KB $\mathcal{K}$, an ABox $\mathcal{A}$, and a signature Σ,
we construct a reachability game (G, F) such that

there exists a Σ-homomorphism from $\mathcal{U}_{\mathcal{K}}$ to $\mathcal{U}_{\mathcal{A}}$

iff

Duplicator has a strategy against Spoiler
to avoid F in G.

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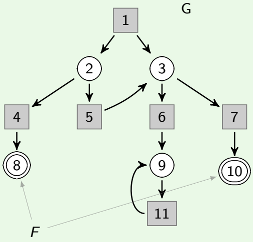
Duplicator wins from 1



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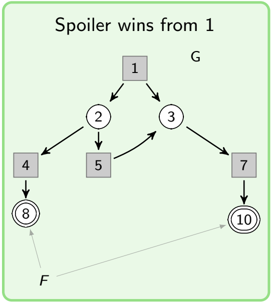

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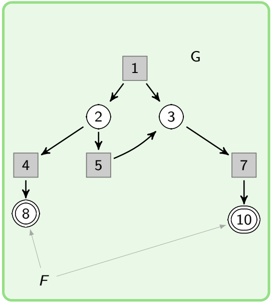
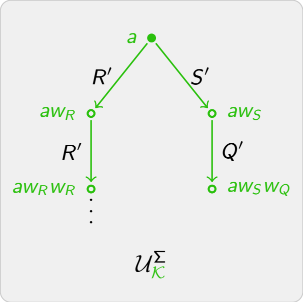
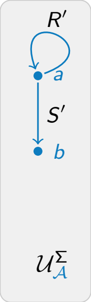
Duplicator has a strategy against Spoiler to avoid F in G. Decidable in PTIME.



$\star$ plus one more condition of no technical interest

❶ Reachability game to check Σ-homomorphism from $\mathcal{U}_\mathcal{K}$ to $\mathcal{U}_\mathcal{A}$





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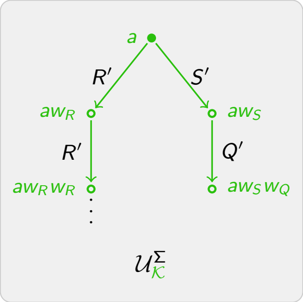

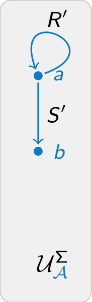

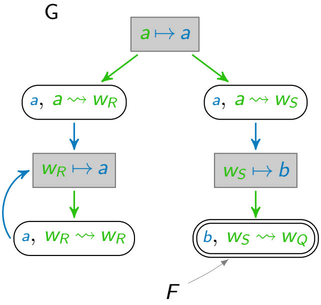

The reachability game $G_\Sigma(\mathcal{G}_\mathcal{K}, \mathcal{U}_\mathcal{A}) = (\mathsf{G}, F)$

Duplicator has a strategy against Spoiler to avoid F in G from $a \mapsto a$ iff

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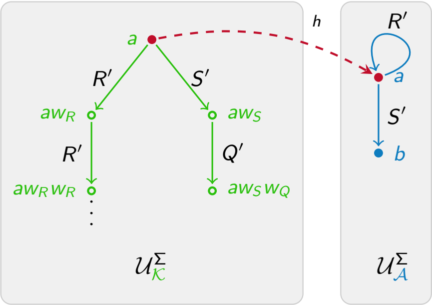
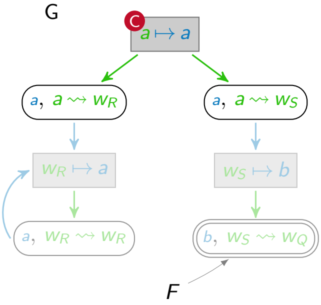
❶ Reachability game to check Σ-homomorphism from $\mathcal{U}_\mathcal{K}$ to $\mathcal{U}_\mathcal{A}$





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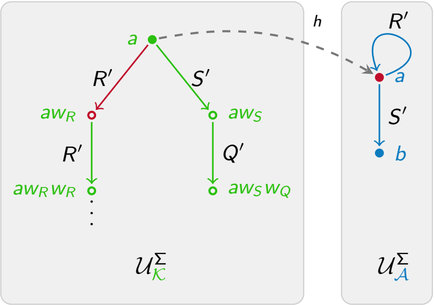

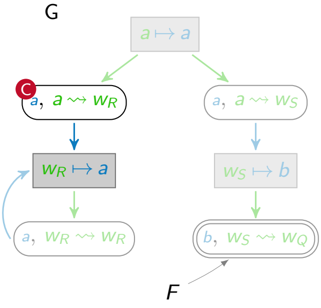

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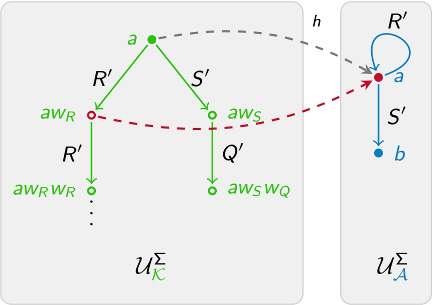

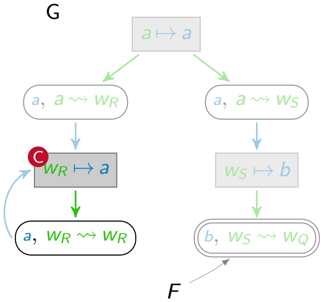

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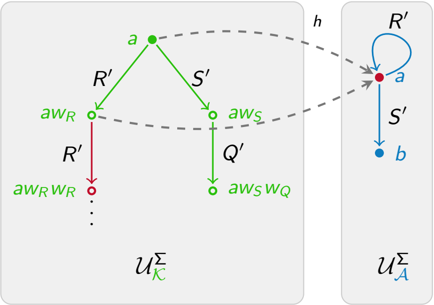

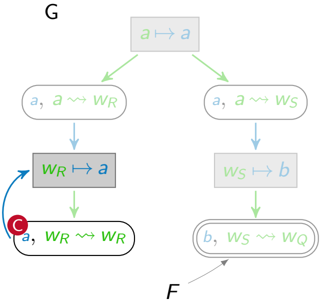

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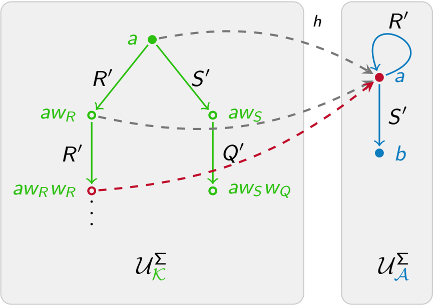

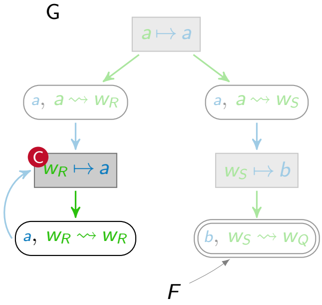

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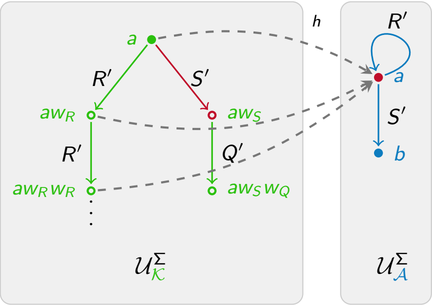

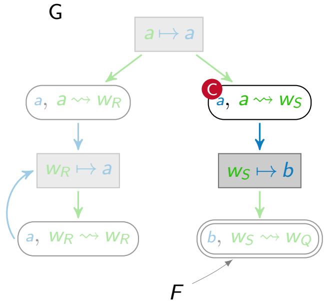

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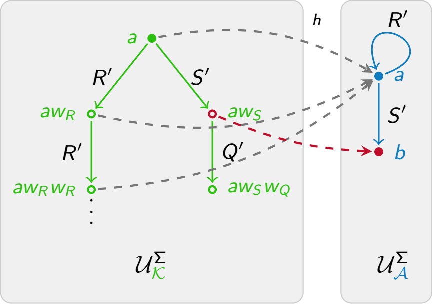

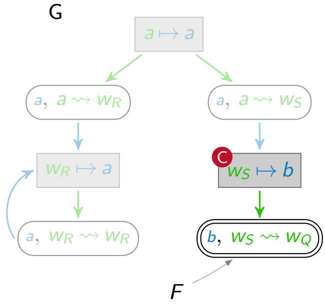

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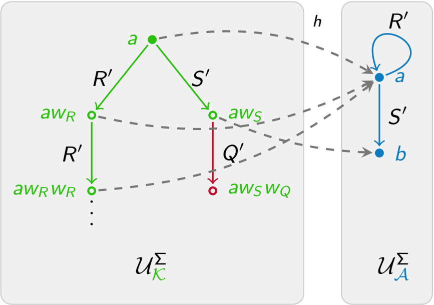

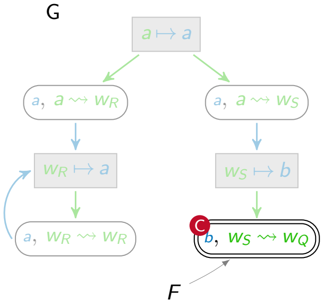

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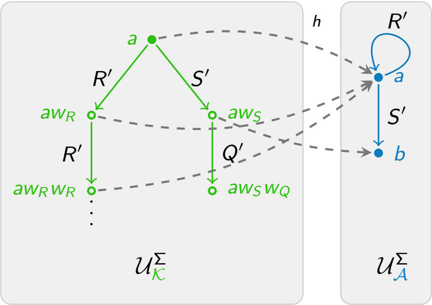
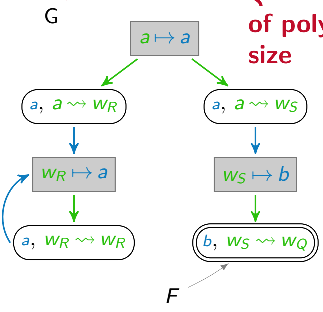


of polynomial size



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Duplicator has a strategy against Spoiler to avoid F in G from $a \mapsto a$ iff

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decidable in polynomial time

Non-emptiness for Extended Universal Solutions is in ExpTime

There exists a universal solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff*
$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ is Σ_2-homomorphically embeddable to a finite subset of itself.

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For KBs $\mathcal{K}_1$, $\mathcal{K}_2$, and a signature Σ, we construct a two-way alternating automaton (2ATA) $\mathbb{A}$ that accepts a tree encoding a finite subset $\mathcal{C}$ of $\mathcal{U}_{\mathcal{K}_2}$ such that

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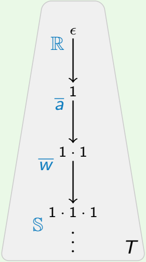
$\mathbb{A}$ accepts trees T of the form

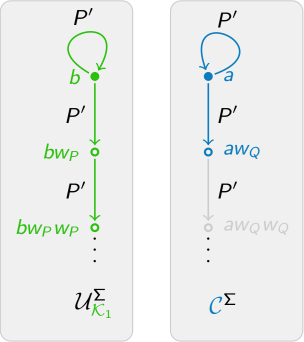


$\mathbb{A}$ "launches" two threads from ϵ:

- one thread verifies that T encodes a finite subset $\mathcal{C}$ of $\mathcal{U}_{\mathcal{K}_2}$ (stops when sees an $\mathbb{S}$).
- the other thread tries to find a Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to $\mathcal{C}$ by traversing T down and up.

for $a \in \mathsf{Ind}(\mathcal{K}_2)$, $w \in \mathsf{Wit}(\mathcal{K}_2)$.

❷ 2ATA $\mathbb{A}$ to check Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to a finite subset $\mathcal{C}$ of $\mathcal{U}_{\mathcal{K}_2}$



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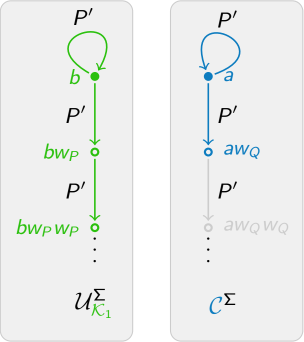

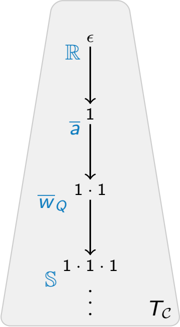

There exists an accepting run of the 2ATA $\mathbb{A}$ over $T_{\mathcal{C}}$ iff

$\mathcal{U}_{\mathcal{K}_1}$ is Σ_2-homomorphically embeddable into $\mathcal{C}$, a finite subset of $\mathcal{U}_{\mathcal{K}_2}$.

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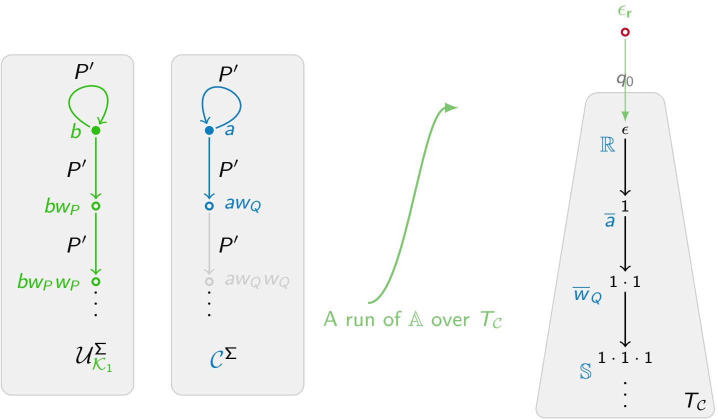

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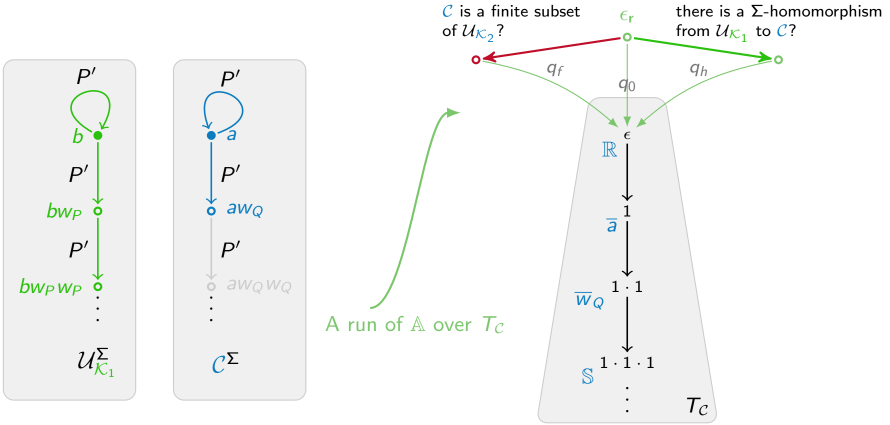

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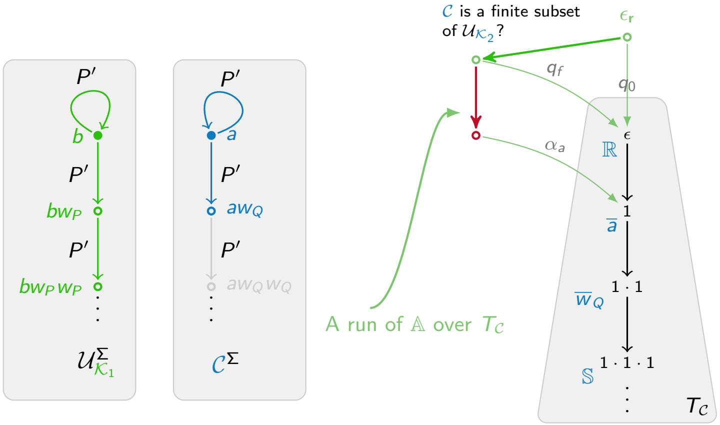

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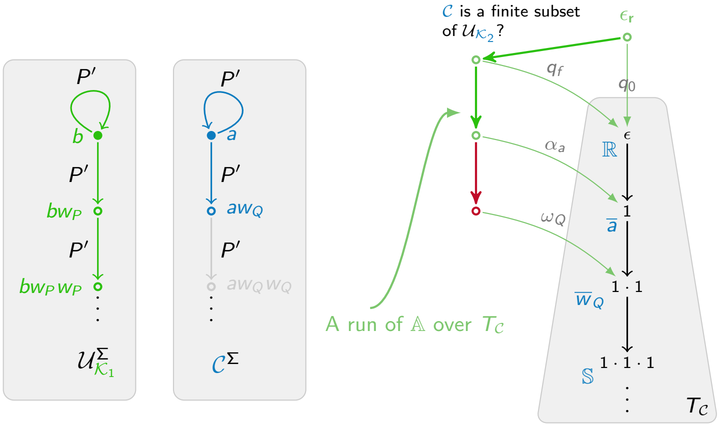

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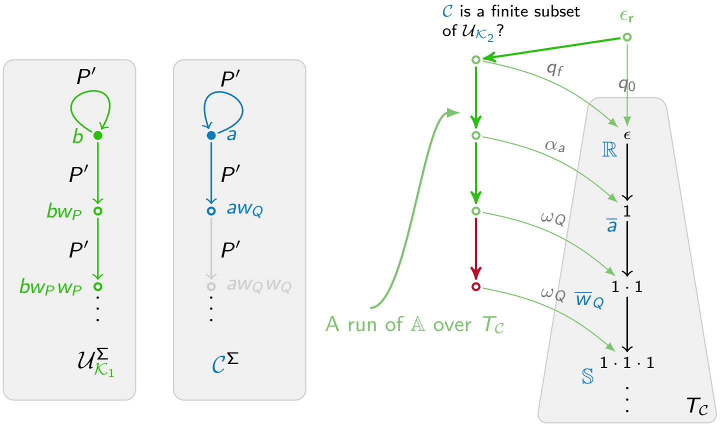

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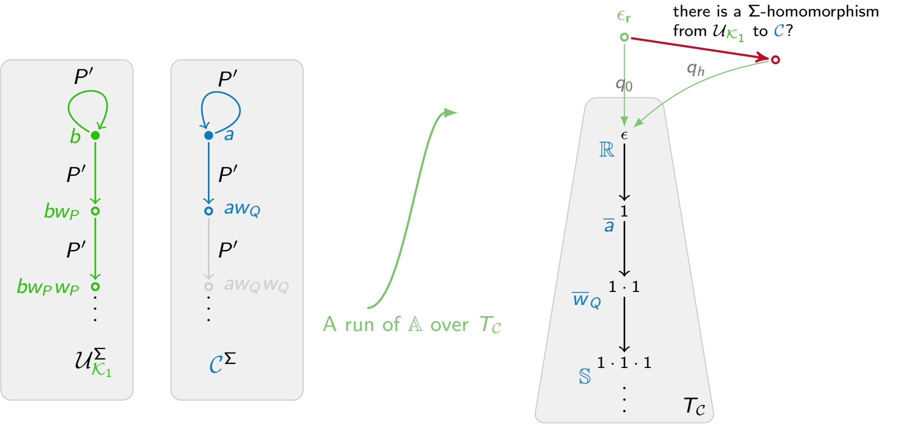

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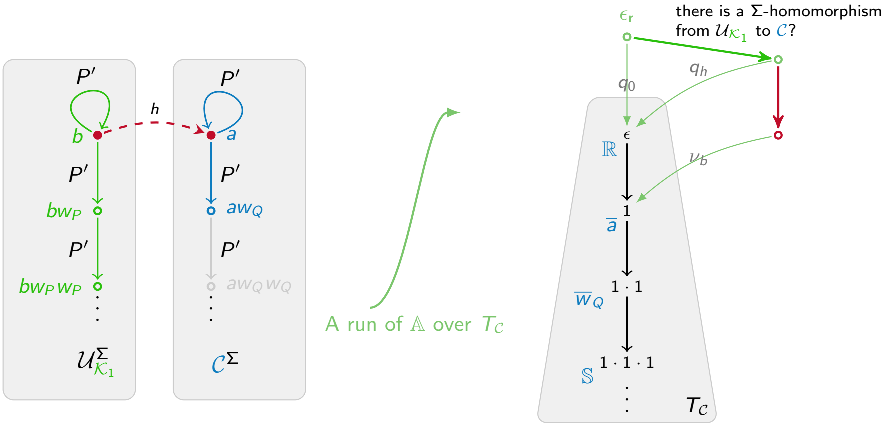

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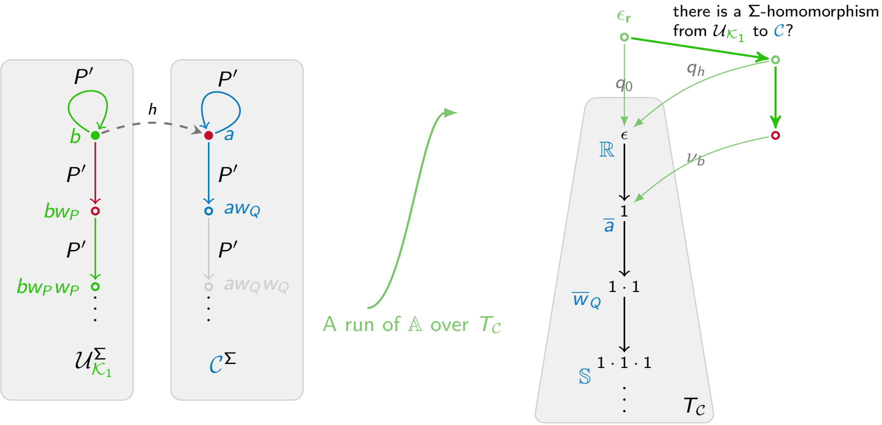

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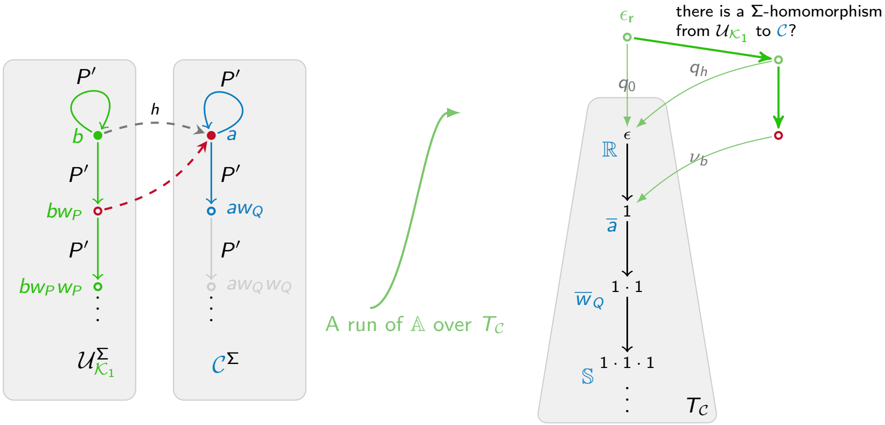

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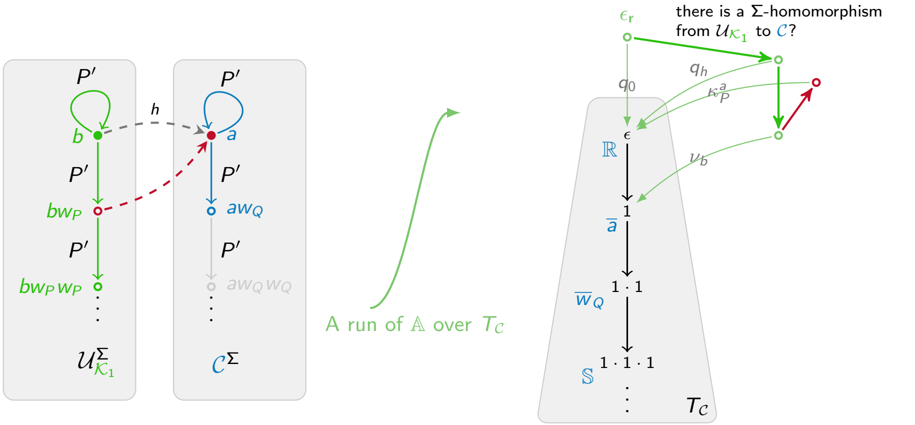

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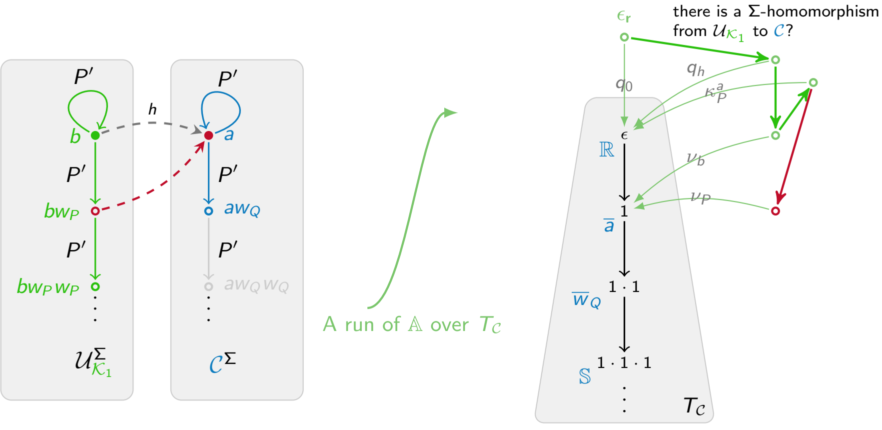

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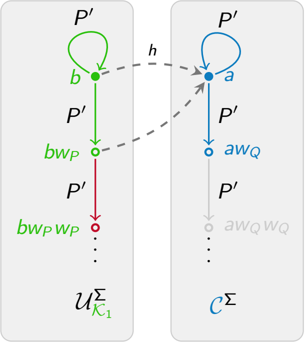
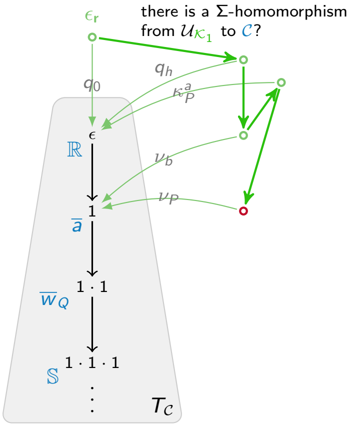


A run of $\mathbb{A}$ over $T_{\mathcal{C}}$



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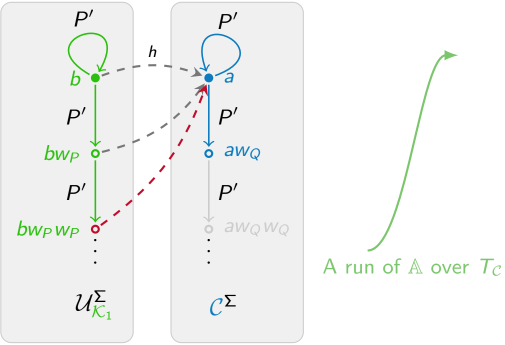

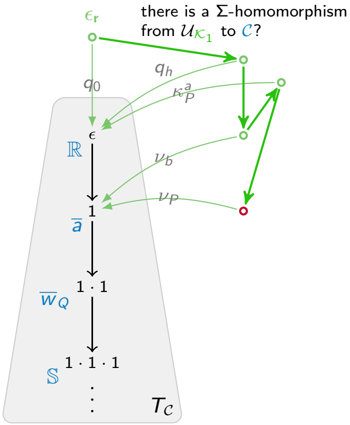

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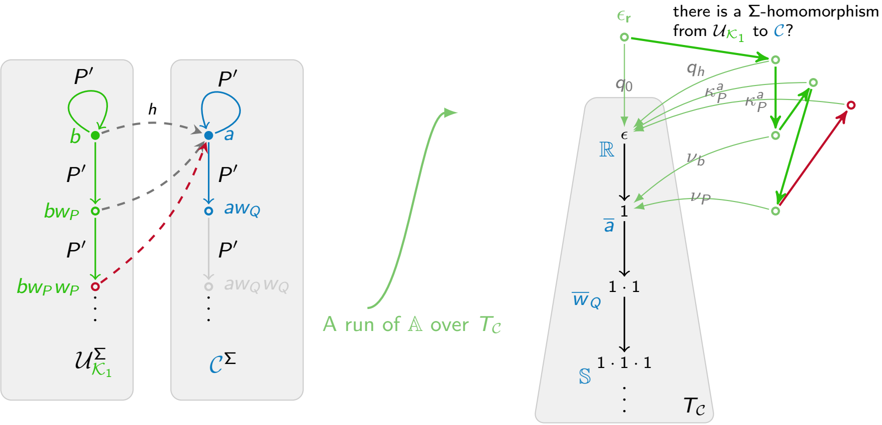

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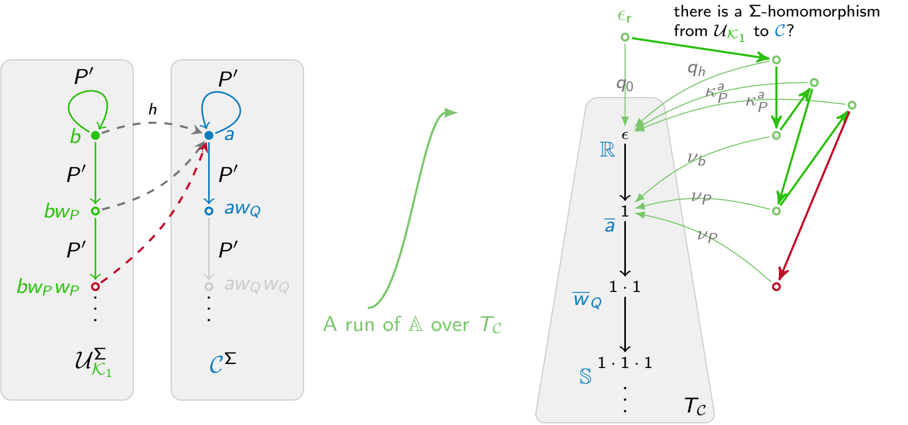

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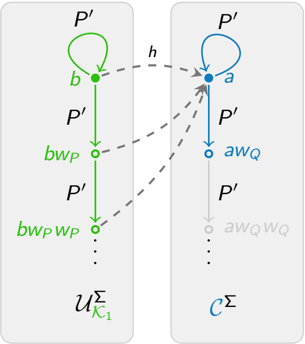
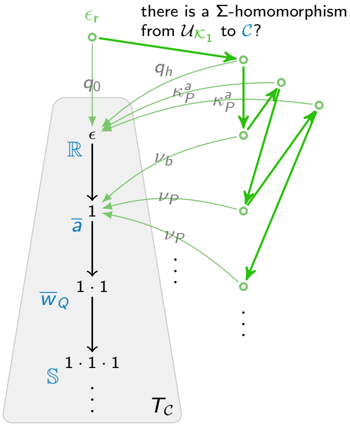


A run of $\mathbb{A}$ over $T_{\mathcal{C}}$



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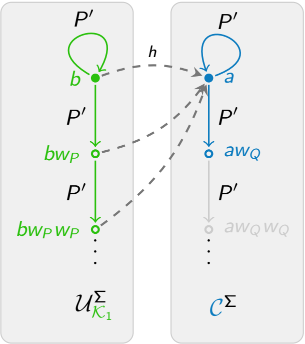

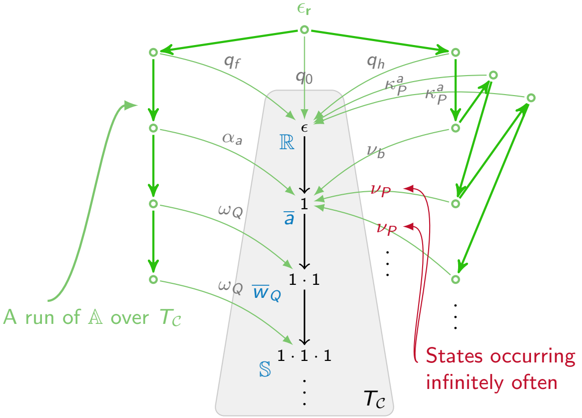

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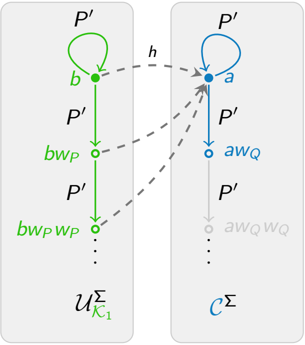

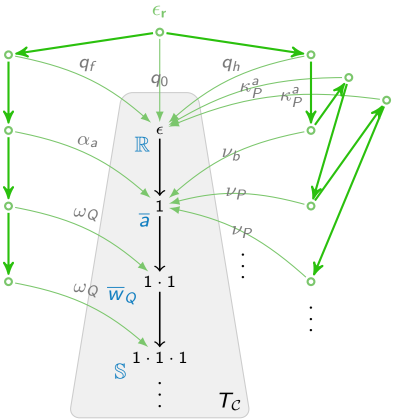

There exists an accepting run of the 2ATA $\mathbb{A}$ over $T_{\mathcal{C}}$ iff

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decidable in exponential time

Outline

1. Introduction
2. Summary of Work
3. Results
4. Technical Development
    - Universal Solutions
    - Universal UCQ-solutions
    - UCQ-representations

Membership for Universal UCQ-Solutions is in ExpTime

$\langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal UCQ-solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff
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For KBs $\mathcal{K}_1$, $\mathcal{K}_2$, and a signature Σ, we use reachability games for checking whether $\mathcal{U}_{\mathcal{K}_1}$ is finitely Σ-homomorphically embeddable into $\mathcal{U}_{\mathcal{K}_2}$.

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Now, $\mathcal{U}_{\mathcal{K}_2}$ is in general infinite.

- we cannot use the game $G_\Sigma(\mathcal{G}_{\mathcal{K}_1}, \mathcal{U}_{\mathcal{K}_2}) = (\mathsf{G}_i, F_i)$, a straightforward extension of $G_\Sigma(\mathcal{G}_{\mathcal{K}}, \mathcal{U}_{\mathcal{A}})$, as G_i is in general infinite.
- so we define a game $G_\Sigma(\mathcal{G}_{\mathcal{K}_1}, \mathcal{G}_{\mathcal{K}_2}) = (\mathsf{G}_f, F_f)$, where G_f is of exponential size and the states have a more complicated structure involving

$$\{u_1, \ldots, u_k\} \mapsto w.$$

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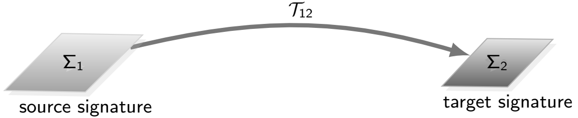
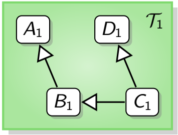
Hence, we obtain an ExpTime upper bound.

Outline

UCQ-representability

mapping $\mathcal{M}$:





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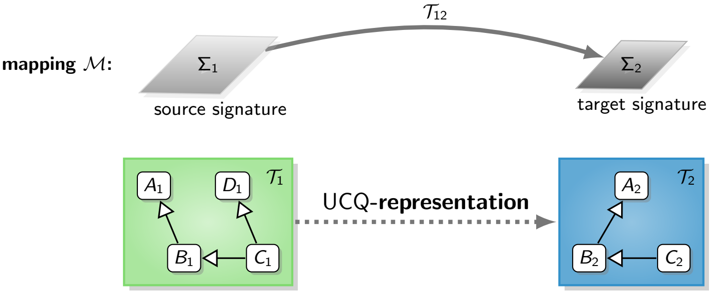

such that for each ABox $\mathcal{A}_1$ and each query q

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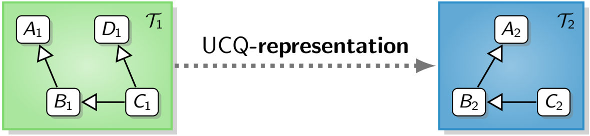

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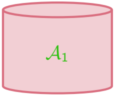

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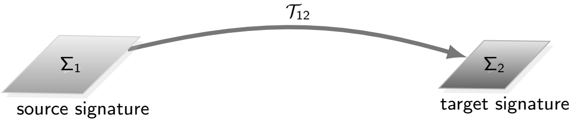

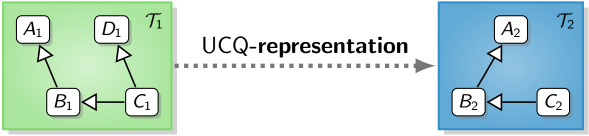

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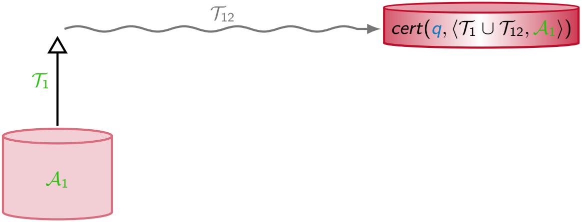

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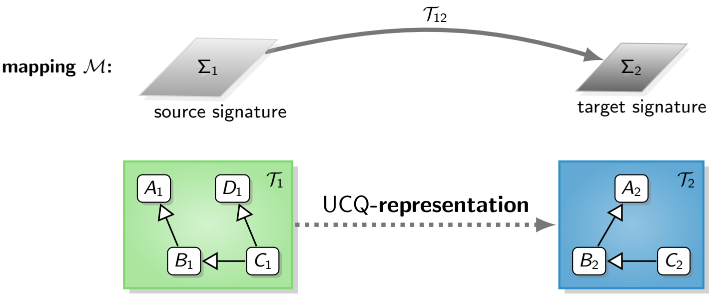

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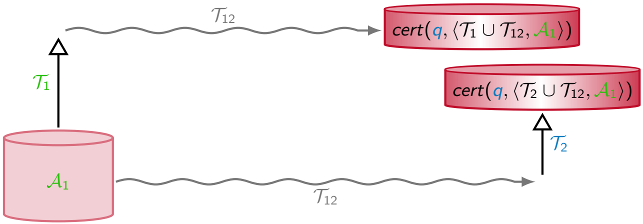

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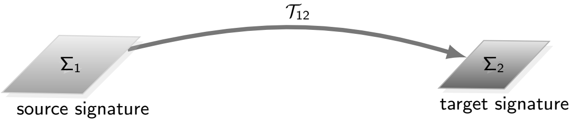

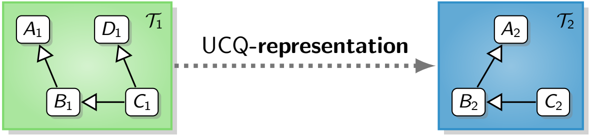

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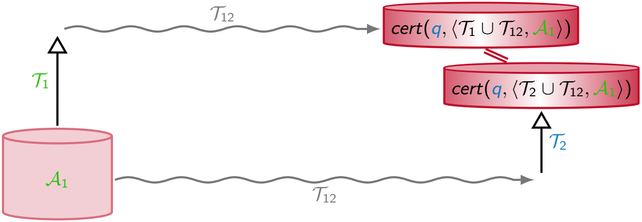

Membership for UCQ-representations is in NLogSpace

$\mathcal{T}_2$ is a UCQ-representation of $\mathcal{T}_1$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff:

Let $\mathcal{A}_1 = \{X(a)\}$.

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$\mathcal{T}_1 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$ $\mathcal{T}_2 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$

1) $a \bullet X, X'$ $\mathcal{G}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ iff $a \bullet X, X'$ $\mathcal{G}_{\langle \mathcal{T}_2 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$

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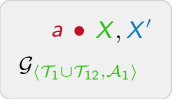
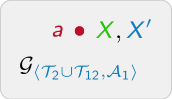
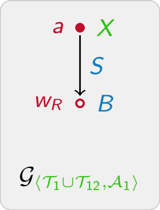
Let $\mathcal{A}_1 = \{X(a)\}$.

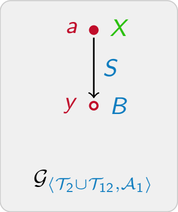
$\mathcal{T}_1 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$ $\qquad$ $\mathcal{T}_2 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$

1) 

iff 

$a \rightsquigarrow_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle} w_R$ $\qquad$ there exists y in $\mathcal{G}_{\langle \mathcal{T}_2 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$:

2) 

$\Rightarrow$ 

Membership for UCQ-representations is in NLogSpace

$\mathcal{T}_2$ is a UCQ-representation of $\mathcal{T}_1$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff:

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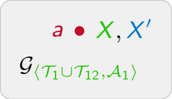
$\mathcal{T}_1 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$ $\quad\quad$ $\mathcal{T}_2 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$

1)



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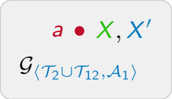
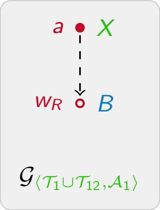
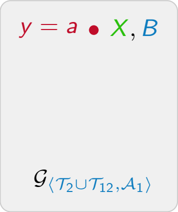


$a \rightsquigarrow_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle} w_R$ $\quad\quad$ there exists y in $\mathcal{G}_{\langle \mathcal{T}_2 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$:

2)



$\Rightarrow$



Membership for UCQ-representations is in NLogSpace

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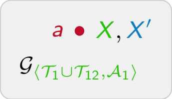
$\mathcal{T}_1 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$ $\qquad\qquad$ $\mathcal{T}_2 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$

1)



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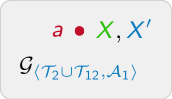
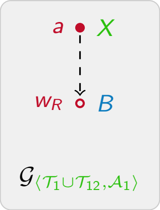
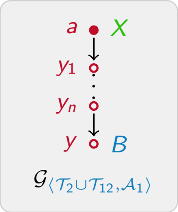


$a \rightsquigarrow_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle} w_R$ $\qquad\qquad$ there exists y in $\mathcal{G}_{\langle \mathcal{T}_2 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$:

2)



$\Rightarrow$



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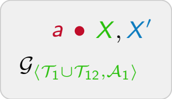
$\mathcal{T}_1 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$ $\qquad$ $\mathcal{T}_2 \cup \mathcal{T}_{12} \models X \sqsubseteq X'$

1)



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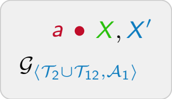
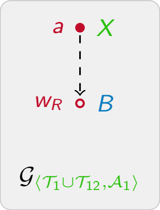
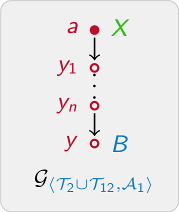


$a \rightsquigarrow_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle} w_R$ $\qquad$ there exists y in $\mathcal{G}_{\langle \mathcal{T}_2 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$:

2)



$\Rightarrow$



Let $\mathcal{A}_1 = \{X(a), Y(a)\}$

3) $\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle$ is inconsistent $\quad$ iff $\quad$ $\langle \mathcal{T}_2 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle$ is inconsistent

Non-emptiness for UCQ-representations is in NLogSpace

Let $\Sigma_1 = \{A(\cdot), B(\cdot), C(\cdot)\}$, $\Sigma_2 = \{A'(\cdot), B'(\cdot), C'(\cdot)\}$, and $\mathcal{T}_1 = \{A \sqsubseteq B\}$.

Is there $\mathcal{T}_2$, a UCQ-representation for $\mathcal{T}_1$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$, where $\mathcal{T}_{12}$ is as follows (gray arrows):

Non-emptiness for UCQ-representations is in NLogSpace

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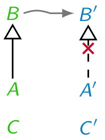

NO

We have that $\mathcal{T}_1 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.
So $\mathcal{T}_2$ should be such that $\mathcal{T}_2 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.

Let $\Sigma_1 = \{A(\cdot), B(\cdot), C(\cdot)\}$, $\Sigma_2 = \{A'(\cdot), B'(\cdot), C'(\cdot)\}$, and $\mathcal{T}_1 = \{A \sqsubseteq B\}$.

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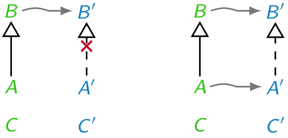

NO YES

We have that $\mathcal{T}_1 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.
So $\mathcal{T}_2$ should be such that $\mathcal{T}_2 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.

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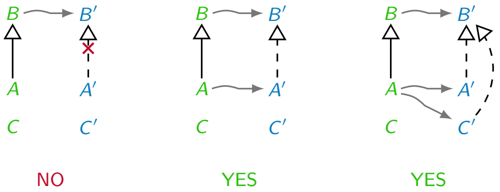

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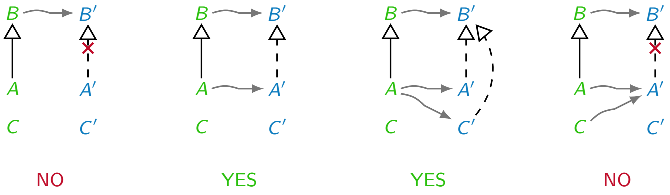
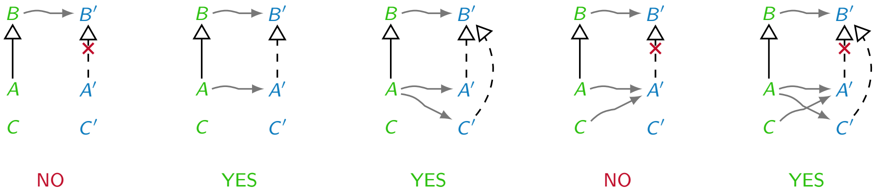


We have that $\mathcal{T}_1 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.
So $\mathcal{T}_2$ should be such that $\mathcal{T}_2 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.

If $\mathcal{T}_2 \cup \mathcal{T}_{12} \models C \sqsubseteq B'$, then it should be the case $\mathcal{T}_1 \cup \mathcal{T}_{12} \models C \sqsubseteq B'$.

Let $\Sigma_1 = \{A(\cdot), B(\cdot), C(\cdot)\}$, $\Sigma_2 = \{A'(\cdot), B'(\cdot), C'(\cdot)\}$, and $\mathcal{T}_1 = \{A \sqsubseteq B\}$.

Is there $\mathcal{T}_2$, a UCQ-representation for $\mathcal{T}_1$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$, where $\mathcal{T}_{12}$ is as follows (gray arrows):



We have that $\mathcal{T}_1 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.
So $\mathcal{T}_2$ should be such that $\mathcal{T}_2 \cup \mathcal{T}_{12} \models A \sqsubseteq B'$.

If $\mathcal{T}_2 \cup \mathcal{T}_{12} \models C \sqsubseteq B'$, then it should be the case $\mathcal{T}_1 \cup \mathcal{T}_{12} \models C \sqsubseteq B'$.

Publications

Conference Publications

- E. Botoeva, R. Kontchakov, V. Ryzhikov, F. Wolter, and M. Zakharyaschev.
  Query inseparability for description logic knowledge bases.
  In *Proc. of the 14th Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR 2014)*. To appear.
- M. Arenas, E. Botoeva, D. Calvanese, and V. Ryzhikov.
  Exchanging OWL 2 QL knowledge bases.
  In *Proc. of the 23rd Int. Joint Conf. on Artificial Intelligence (IJCAI 2013)*, pages 703-710, 2013.
- M. Arenas, E. Botoeva, D. Calvanese, V. Ryzhikov, and E. Sherkhonov.
  Exchanging description logic knowledge bases.
  In *Proc. of the 13th Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR 2012)*, pages 563-567.

Workshop Publications

- M. Arenas, E. Botoeva, D. Calvanese, and V. Ryzhikov.
  Computing solutions in OWL 2 QL knowledge exchange.
  In *Proc. of the 26th Int. Workshop on Description Logic (DL 2013)*, volume 1014, pages 4-16, 2013.
- M. Arenas, E. Botoeva, D. Calvanese, V. Ryzhikov, and E. Sherkhonov.
  Representability in DL-Liter knowledge base exchange.
  In *Proc. of the 25th Int. Workshop on Description Logic (DL 2012)*, volume 846, 2012.
- M. Arenas, E. Botoeva, and D. Calvanese.
  Knowledge base exchange.
  In *Proc. of the 24th Int. Workshop on Description Logic (DL 2011)*, volume 745, 2011.

Publications cont.

Technical Reports

- M. Arenas, E. Botoeva, D. Calvanese, and V. Ryzhikov.
  Exchanging OWL 2 QL knowledge bases (extended version).
  CoRR Technical Report arXiv:1304.5810, arXiv.org e-Print archive, 2013. Available at http://arxiv.org/abs/1304.5810.

Under Submission

- M. Arenas, E. Botoeva, D. Calvanese, and V. Ryzhikov.
  Knowledge base exchange: The case of OWL 2 QL.
  Under submission to a journal.

Thank you for your attention!

Universal solutions	simple ABoxes	extended ABoxes
Membership	PTIME-complete	NP-complete
Non-emptiness	PTIME-complete	PSPACE-hard, in EXPTIME

Universal UCQ-**solutions**	simple ABoxes	extended ABoxes
Membership	PSPACE-hard	in EXPTIME
Non-emptiness	in EXPTIME	PSPACE-hard

UCQ-**representations**	Complexity
Membership	NLOGSPACE-complete
Non-emptiness	NLOGSPACE-complete

Marcelo Arenas, Jorge Pérez, and Juan L. Reutter. Data exchange beyond complete data. pages 83–94, 2011.

Ronald Fagin, Phokion G. Kolaitis, Renée J. Miller, and Lucian Popa. Data exchange: Semantics and query answering. pages 207–224, 2003.

Knowledge Base Exchange: Example

$$\mathcal{M}: \begin{array}{lcl} \exists AuthorOf^- & \sqsubseteq & \exists BookGenre \\ AuthorOf^- & \sqsubseteq & WrittenBy \\ TaxNumber & \sqsubseteq & SSN \end{array}$$

$$\mathcal{T}_1: \begin{array}{l} \exists AuthorOf \sqsubseteq Author \\ Author \sqsubseteq \exists TaxNumber \end{array}$$

$\mathcal{A}_1$:

AuthorOf	
nabokov	lolita
tolkien	lotr

Knowledge Base Exchange: Example

$\mathcal{M}$:

$$\exists AuthorOf^- \sqsubseteq \exists BookGenre$$
$$AuthorOf^- \sqsubseteq WrittenBy$$
$$TaxNumber \sqsubseteq SSN$$

$\mathcal{T}_1$:

$$\exists AuthorOf \sqsubseteq Author$$
$$Author \sqsubseteq \exists TaxNumber$$

$\mathcal{A}_1$:

AuthorOf	
nabokov	lolita
tolkien	lotr

$\mathcal{A}_2$:

WrittenBy	
lolita	nabokov
lotr	tolkien

SSN	
nabokov	$null_1$
tolkien	$null_2$

BookGenre	
lolita	$null_3$
lotr	$null_4$

$\mathcal{A}_2$ is a universal solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ (with extended ABoxes).

Knowledge Base Exchange: Example

$$\mathcal{M}: \begin{array}{lcl} \exists AuthorOf^- & \sqsubseteq & \exists BookGenre \\ AuthorOf^- & \sqsubseteq & WrittenBy \\ TaxNumber & \sqsubseteq & SSN \end{array}$$

$$\mathcal{T}_1: \begin{array}{l} \exists AuthorOf \sqsubseteq Author \\ Author \sqsubseteq \exists TaxNumber \end{array} \qquad \mathcal{T}_2: \begin{array}{l} \exists WrittenBy^- \sqsubseteq \exists SSN \\ \exists WrittenBy \sqsubseteq \exists BookGenre \end{array}$$

$\mathcal{A}_1$:

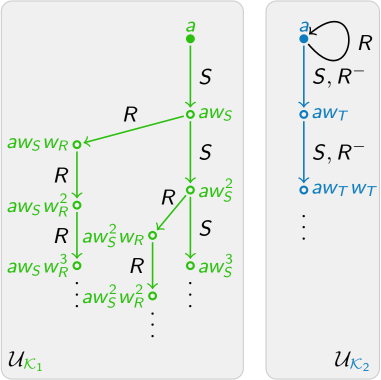
AuthorOf	
nabokov	lolita
tolkien	lotr

$\mathcal{A}_2$:

WrittenBy	
lolita	nabokov
lotr	tolkien

$\langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal-UCQ solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ (with simple ABoxes).

❸ Reachability game to check Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to $\mathcal{U}_{\mathcal{K}_2}$



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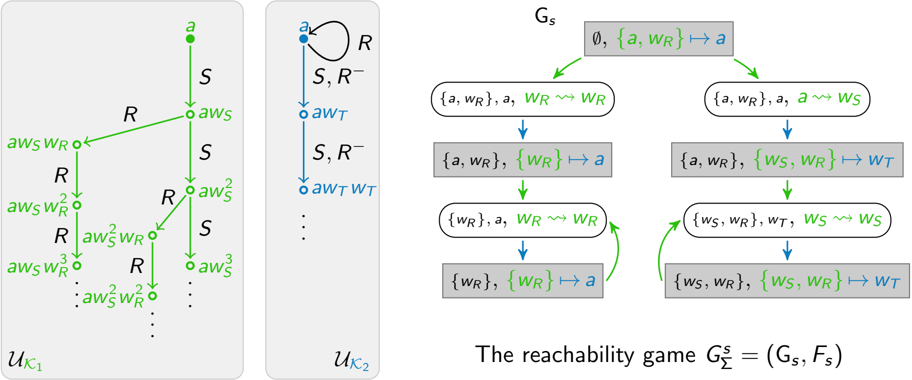

The reachability game $G^s_\Sigma = (\mathsf{G}_s, F_s)$

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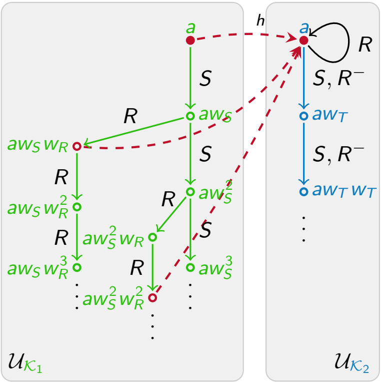

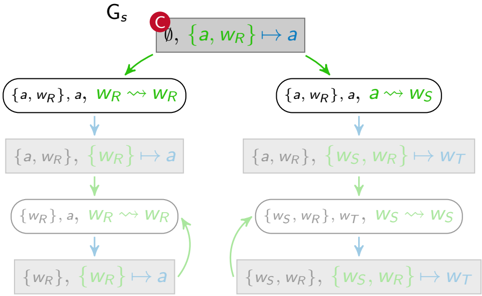

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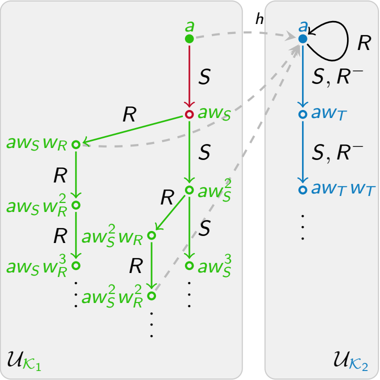

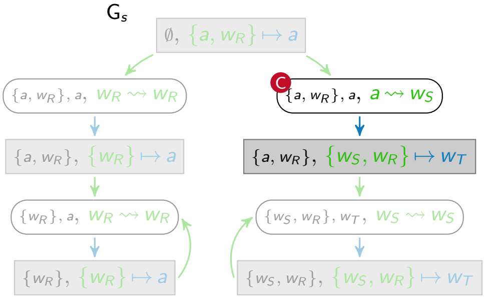

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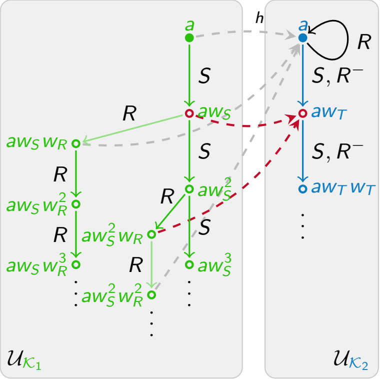

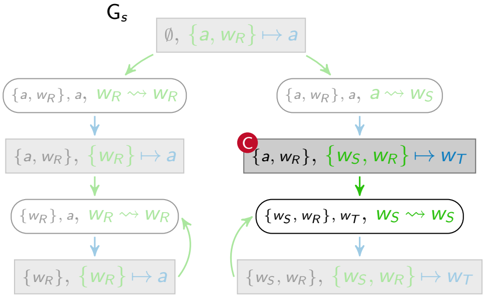

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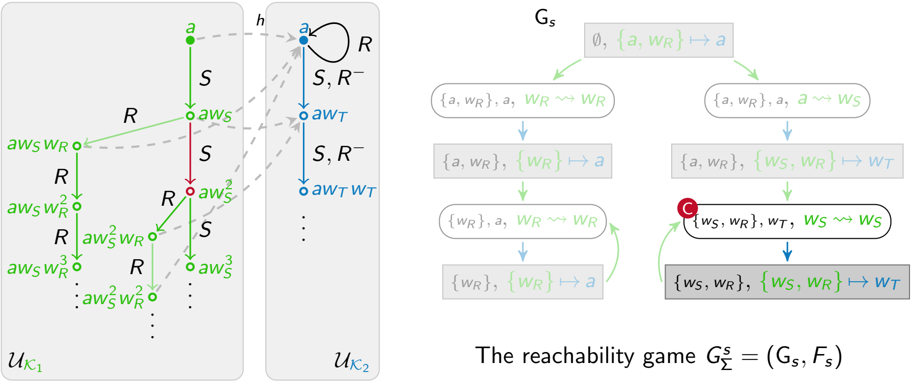

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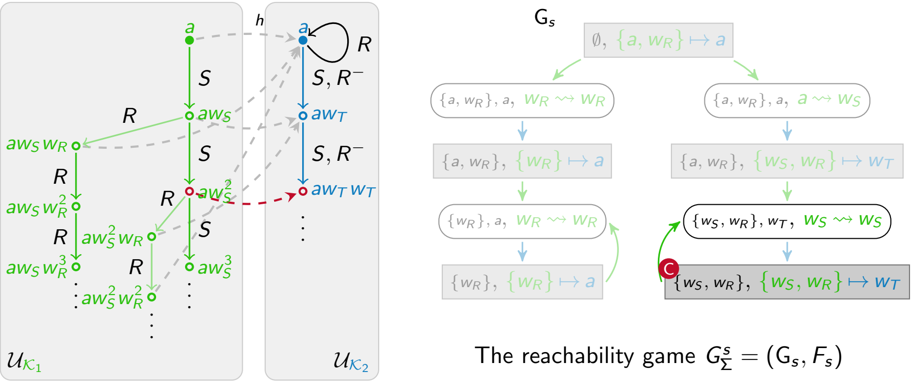

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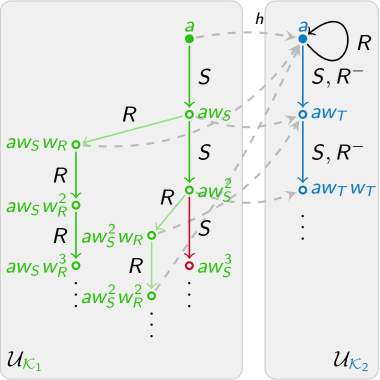

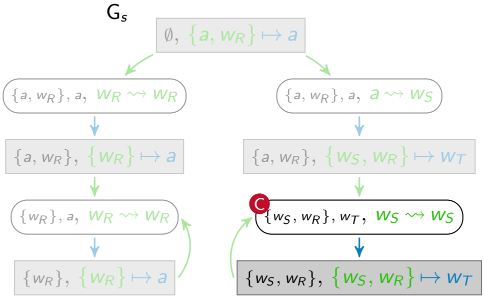

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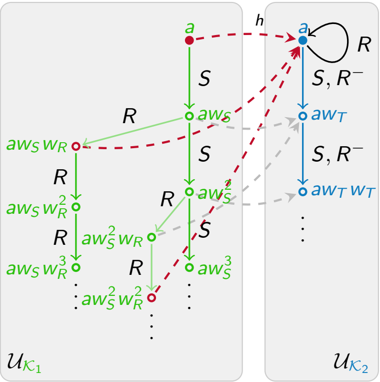

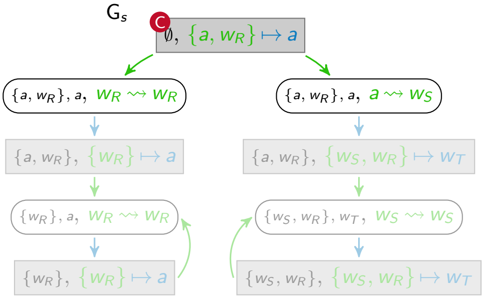

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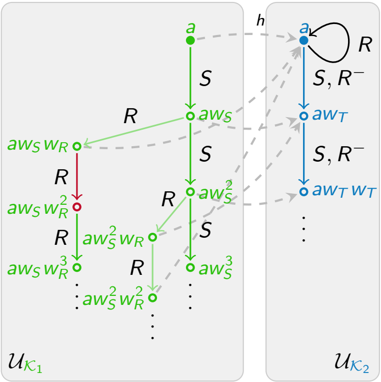
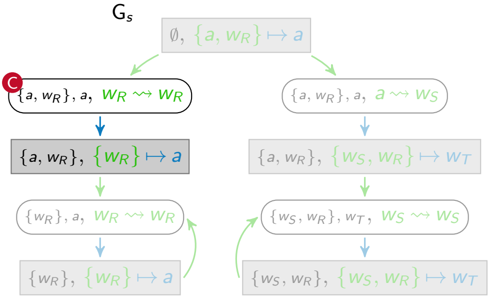
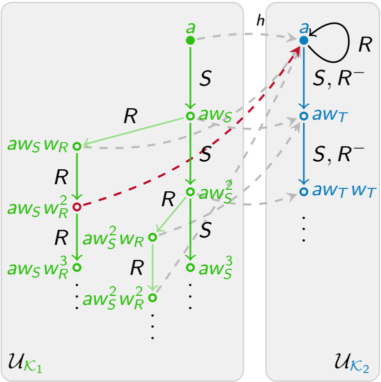
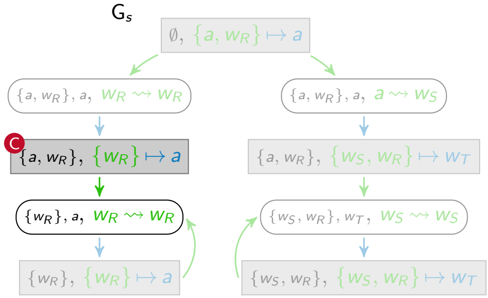




The reachability game $G^s_\Sigma = (\mathsf{G}_s, F_s)$

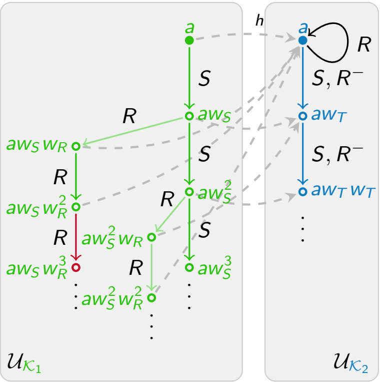
3 Reachability game to check Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to $\mathcal{U}_{\mathcal{K}_2}$

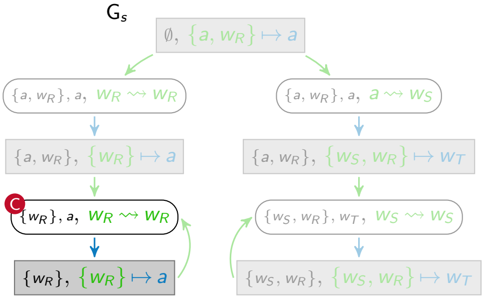




The reachability game $G_\Sigma^s = (\mathsf{G}_s, F_s)$

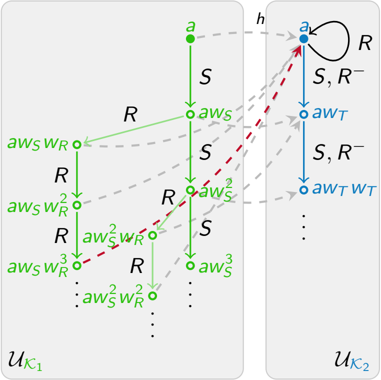
❸ Reachability game to check Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to $\mathcal{U}_{\mathcal{K}_2}$

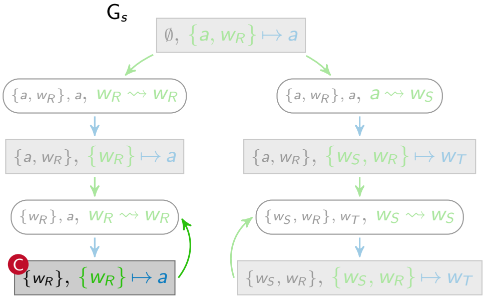




The reachability game $G^s_\Sigma = (\mathsf{G}_s, F_s)$

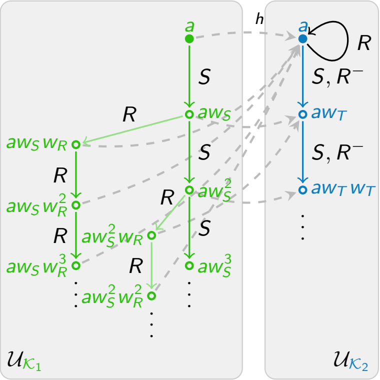
❸ Reachability game to check Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to $\mathcal{U}_{\mathcal{K}_2}$

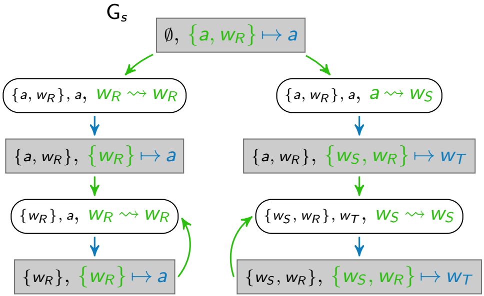




The reachability game $G_\Sigma^s = (\mathsf{G}_s, F_s)$

❸ Reachability game to check Σ-homomorphism from $\mathcal{U}_{\mathcal{K}_1}$ to $\mathcal{U}_{\mathcal{K}_2}$





The reachability game $G^s_\Sigma = (\mathsf{G}_s, F_s)$