Program Analysis (70020)

While Language

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Syntactic Constructs

We use the following syntactic categories:

a	$\in$	**AExp**	arithmetic expressions
b	$\in$	**BExp**	boolean expressions
S	$\in$	**Stmt**	statements

Abstract Syntax of WHILE

The syntax of the language WHILE is given by the following **abstract syntax**:

a

b

S

Abstract Syntax of WHILE

The syntax of the language WHILE is given by the following **abstract syntax**:

$$a ::= x$$

b

S

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$$
\begin{array}{lcl}
a & ::= & x \mid n \\
b & & \\
S & &
\end{array}
$$

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The syntax of the language WHILE is given by the following **abstract syntax**:

$$a \quad ::= \quad x \mid n \mid a_1 \; op_a \; a_2$$

$$b$$

$$S$$

Abstract Syntax of WHILE

The syntax of the language WHILE is given by the following **abstract syntax**:

$$a \quad ::= \quad x \mid n \mid a_1 \ op_a \ a_2$$

$$b \quad ::= \quad true$$

$$S$$

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S &::= x := a
\end{aligned}
$$

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\end{array}
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Syntactical Categories

We assume some countable/finite set of variables is given;

$$\begin{array}{rcll} x, y, z, \ldots & \in & \textbf{Var} & \text{variables} \\ n, m, \ldots & \in & \textbf{Num} & \text{numerals} \end{array}$$

Syntactical Categories

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$x, y, z, \ldots$	$\in$	**Var**	variables
$n, m, \ldots$	$\in$	**Num**	numerals
$\ell, \ldots$	$\in$	**Lab**	labels

Numerals (integer constants) will not be further defined and neither will the operators:

op_a	$\in$	$\mathbf{Op}_a$	arithmetic operators, e.g. $+, -, \times, \ldots$
op_b	$\in$	$\mathbf{Op}_b$	boolean operators, e.g. $\wedge, \vee, \ldots$
op_r	$\in$	$\mathbf{Op}_r$	relational operators, e.g. $=, <, \leq, \ldots$

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An Example in WHILE

An example of a program written in this WHILE language is the following one which computes the factorial of the number stored in x and leaves the result in z:

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[\, z := 1 \,]^2; \\
\textbf{while } [y > 1]^3 \textbf{ do } (\\
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Note the use of meta-symbols, brackets, to group statements.

Concrete Syntax of WHILE

To avoid using brackets (as meta-symbols) we could also use the **concrete syntax** of the language WHILE as follows:

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S

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 & & \mid \textbf{while} \ b \ \textbf{do} \ S \ \textbf{od}
\end{array}
$$

Initial Label

When presenting examples of Data Flow Analyses we will use a number of operations on programs and labels. The first of these is

$$init : \textbf{Stmt} \rightarrow \textbf{Lab}$$

which returns the initial label of a statement:

$$\begin{aligned}
init([\, x := a\,]^{\ell}) &= \ell \\
init([\, \textbf{skip}\,]^{\ell}) &= \ell \\
init(S_1;S_2) &= init(S_1) \\
init(\textbf{if } [b]^{\ell} \textbf{ then } S_1 \textbf{ else } S_2) &= \ell \\
init(\textbf{while } [b]^{\ell} \textbf{ do } S) &= \ell
\end{aligned}$$

Final Labels

We will also need a function which returns the set of final labels in a statement; whereas a sequence of statements has a single entry, it may have multiple exits (e.g. in the conditional):

$$\mathit{final} : \textbf{Stmt} \to \mathcal{P}(\textbf{Lab})$$

$$\begin{aligned}
\mathit{final}([\, x := a \,]^{\ell}) &= \{\ell\} \\
\mathit{final}([\, \textbf{skip} \,]^{\ell}) &= \{\ell\} \\
\mathit{final}(S_1 ; S_2) &= \mathit{final}(S_2) \\
\mathit{final}(\textbf{if } [b]^{\ell} \textbf{ then } S_1 \textbf{ else } S_2) &= \mathit{final}(S_1) \cup \mathit{final}(S_2) \\
\mathit{final}(\textbf{while } [b]^{\ell} \textbf{ do } S) &= \{\ell\}
\end{aligned}$$

The **while**-loop terminates immediately after the test fails.

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- tests of the form $[b]^{\ell}$.

Blocks

To access the statements or test associated with a label in a program we use the function

$$blocks : \textbf{Stmt} \rightarrow \mathcal{P}(\textbf{Block})$$

$$\begin{array}{rcl}
blocks([\, x := a \,]^{\ell}) & = & \{[\, x := a \,]^{\ell}\} \\
blocks([\, \textbf{skip} \,]^{\ell}) & = & \{[\, \textbf{skip} \,]^{\ell}\} \\
blocks(S_1 ; S_2) & = & blocks(S_1) \cup blocks(S_2) \\
blocks(\textbf{if } [b]^{\ell} \textbf{ then } S_1 \textbf{ else } S_2) & = & \{[b]^{\ell}\} \cup \\
 & & blocks(S_1) \cup blocks(S_2) \\
blocks(\textbf{while } [b]^{\ell} \textbf{ do } S) & = & \{[b]^{\ell}\} \cup blocks(S)
\end{array}$$

Labels

Then the set of labels occurring in a program is given by

$$\mathit{labels} : \textbf{Stmt} \rightarrow \mathcal{P}(\textbf{Lab})$$

where

$$\mathit{labels}(S) = \{\ell \mid [B]^{\ell} \in \mathit{blocks}(S)\}$$

Clearly $\mathit{init}(S) \in \mathit{labels}(S)$ and $\mathit{final}(S) \subseteq \mathit{labels}(S)$.

Flow

$$\mathit{flow} : \mathbf{Stmt} \rightarrow \mathcal{P}(\mathbf{Lab} \times \mathbf{Lab})$$

which maps statements to sets of flows:

$$\begin{aligned}
\mathit{flow}([\, x := a\,]^{\ell}) &= \emptyset \\
\mathit{flow}([\, \mathbf{skip}\,]^{\ell}) &= \emptyset \\
\mathit{flow}(S_1 ; S_2) &= \mathit{flow}(S_1) \cup \mathit{flow}(S_2) \cup \\
&\quad \{(\ell, \mathit{init}(S_2)) \mid \ell \in \mathit{final}(S_1)\} \\
\mathit{flow}(\mathbf{if}\ [b]^{\ell}\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2) &= \mathit{flow}(S_1) \cup \mathit{flow}(S_2) \cup \\
&\quad \{(\ell, \mathit{init}(S_1)), (\ell, \mathit{init}(S_2))\} \\
\mathit{flow}(\mathbf{while}\ [b]^{\ell}\ \mathbf{do}\ S) &= \mathit{flow}(S) \cup \{(\ell, \mathit{init}(S))\} \cup \\
&\quad \{(\ell', \ell) \mid \ell' \in \mathit{final}(S)\}
\end{aligned}$$

An Example Flow

Consider the following program, power, computing the x-th power of the number stored in y:

$$
\begin{array}{l}
[\ z := 1\ ]^1; \\
\textbf{while}\ [x > 1]^2\ \textbf{do}\ (\\
\qquad [\ z := z * y\ ]^3; \\
\qquad [\ x := x - 1\ ]^4)
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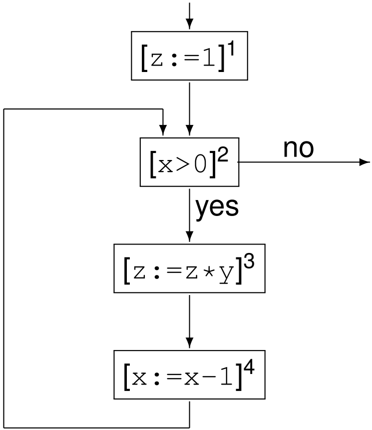
We have *labels*(power) $= \{1, 2, 3, 4\}$, *init*(power) $= 1$, and *final*(power) $= \{2\}$. The function *flow* produces the set:

$$\mathit{flow}(\text{power}) = \{(1, 2), (2, 3), (3, 4), (4, 2)\}$$

Flow Graph



Forward Analysis

The function *flow* is used in the formulation of *forward analyses*. Clearly *init*(S) is the (unique) entry node for the flow graph with nodes *labels*(S) and edges *flow*(S). Also

$$\begin{aligned} \mathit{labels}(S) &= \{\mathit{init}(S)\} \cup \\ &\quad \{\ell \mid (\ell, \ell') \in \mathit{flow}(S)\} \cup \\ &\quad \{\ell' \mid (\ell, \ell') \in \mathit{flow}(S)\} \end{aligned}$$

and for composite statements (meaning those not simply of the form $[B]^{\ell}$) the equation remains true when removing the $\{\mathit{init}(S)\}$ component.

Reverse Flow

In order to formulate *backward analyses* we require a function that computes reverse flows:

$$\mathit{flow}^R : \textbf{Stmt} \rightarrow \mathcal{P}(\textbf{Lab} \times \textbf{Lab})$$

$$\mathit{flow}^R(S) = \{(\ell, \ell') \mid (\ell', \ell) \in \mathit{flow}(S)\}$$

For the power program, flow^R produces

$$\{(2,1), (2,4), (3,2), (4,3)\}$$

Backward Analysis

In case *final*(S) contains just one element that will be the unique entry node for the flow graph with nodes *labels*(S) and edges $\mathit{flow}^R(S)$. Also

$$\begin{aligned} \mathit{labels}(S) &= \mathit{final}(S) \cup \\ &\quad \{\ell \mid (\ell, \ell') \in \mathit{flow}^R(S)\} \cup \\ &\quad \{\ell' \mid (\ell, \ell') \in \mathit{flow}^R(S)\} \end{aligned}$$

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An expression is trivial if it is a single variable or constant.

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We will use the notation $S_\star$ to represent the program we are analysing (the “top-level" statement) and furthermore:

- **Lab**$_\star$ to represent the labels ($labels(S_\star)$) appearing in $S_\star$,
- **Var**$_\star$ to represent the variables ($FV(S_\star)$) appearing in $S_\star$,
- **Block**$_\star$ to represent the elementary blocks ($blocks(S_\star)$) occurring in $S_\star$, and
- **AExp**$_\star$ to represent the set of *non-trivial* arithmetic subexpressions in $S_\star$ as well as
- **AExp**(a) and **AExp**(b) to refer to the set of non-trivial arithmetic subexpressions of a given arithmetic, respectively boolean, expression.

An expression is trivial if it is a single variable or constant.

Isolated Entries & Exits

Program $S_\star$ has *isolated entries* if:

$$\forall \ell \in \mathbf{Lab} : (\ell, \mathit{init}(S_\star)) \notin \mathit{flow}(S_\star)$$

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Similarly, we shall frequently assume that the program $S_\star$ has *isolated exits*; this means that:

$$\forall \ell_1 \in \mathit{final}(S_\star)\ \forall \ell_2 \in \mathbf{Lab} : (\ell_1, \ell_2) \notin \mathit{flow}(S_\star)$$

Label Consistency

A statement, S, is label consistent if and only if:

$$[B_1]^\ell, [B_2]^\ell \in \mathit{blocks}(S) \text{ implies } B_1 = B_2$$

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Clearly, if all blocks in S are uniquely labelled (meaning that each label occurs only once), then S is label consistent.

When S is label consistent the statement or clause "where $[B]^\ell \in \mathit{blocks}(S)$" is unambiguous in defining a partial function from labels to elementary blocks; we shall then say that ℓ labels the block B.